

# Contents

<b>1 Introduction</b> .....	1
1.1. Classical Serre–Tate ordinary theory .....	3
1.2. Translation of the classical theory into our language .....	5
1.3. Generalized language .....	11
1.4. Main results on Shimura $p$ -divisible objects .....	17
1.5. Standard Hodge situations .....	20
1.6. Main results on standard Hodge situations .....	27
1.7. On applications .....	29
1.8. Extra literature .....	31
1.9. Our main motivation .....	32
1.10. More on the overall organization .....	33
<b>2 Preliminaries</b> .....	35
2.1. Standard notations for the abstract theory .....	38
2.2. Descending slope filtration lemma .....	39
2.3. Elementary properties of reductive group schemes .....	40
2.4. Natural $\mathbf{Z}_p$ structures and applications .....	47
2.5. Lie $F$ -isocrystals and parabolic subgroup schemes .....	49
2.6. Ten group schemes .....	51
2.7. Opposition properties .....	54
2.8. Elementary crystalline complements .....	56
<b>3 Abstract theory, I: Proof of Theorem 1.4.1</b> .....	61
3.1. On Hodge cocharacters .....	61
3.2. A general outline .....	66
3.3. The essence of the proofs of Theorems 1.4.1 and 3.2.4 .....	69
3.4. The split, adjoint, cyclic context .....	70
3.5. The proof of the property 3.3 (a) .....	78

3.6. Formulas for group normalizers .....	86
3.7. The proof of the property 3.3 (b) .....	90
3.8. End of the proofs of Theorems 1.4.1 and 3.2.4 .....	93
3.9. On Hasse–Witt invariants .....	99
<b>4 Abstract theory, II: Proofs of Theorems 1.4.2 and 1.4.3</b> .....	101
4.1. Proof of Theorem 1.4.2 .....	101
4.2. Proofs of Theorem 1.4.3 and Corollary 1.4.4 .....	106
4.3. Complements .....	109
<b>5 Abstract theory, III: Formal Lie groups and nilpotent group schemes</b> .....	115
5.1. Sign $p$ -divisible groups .....	115
5.2. Minimal decompositions .....	117
5.3. Group structures: The adjoint, cyclic case .....	126
5.4. Group structures: The general case .....	134
<b>6 Geometric theory: Proofs of the Theorems 1.6.1 to 1.6.3</b> .....	137
6.1. Fontaine comparison theory .....	138
6.2. Deformation theory .....	143
6.3. Proof of Theorem 1.6.1 .....	154
6.4. The existence of geometric lifts .....	157
6.5. Proofs of Theorems 1.6.2 and 1.6.3 .....	162
<b>7 Examples and complements</b> .....	171
7.1. On the Newton polygon $\mathfrak{N}_0$ .....	172
7.2. Modulo $p$ interpretation .....	173
7.3. Examples on $p$ -ranks .....	175
7.4. On the Sh-ordinary locus $\mathcal{O}$ .....	178
7.5. Example for Uni-ordinariness .....	179
<b>8 Two special applications</b> .....	183
8.1. Applications to a conjecture of Milne .....	183
8.2. Applications to integral canonical models .....	192
<b>9 Generalization of the properties 1.1(f) to (i) to the context of standard Hodge situations</b> .....	197
9.1. Generalization of the property 1.1(f) .....	198
9.2. Generalization of the property 1.1(g) .....	204

9.3. Basic notations and definitions.....	206
9.4. Theorem (the existence of canonical formal Lie group structures for $\mathfrak{J}_0 \leq 1$ ).....	208
9.5. Complements .....	213
9.6. Ramified valued points.....	218
9.7. A functorial property .....	223
9.8. Four operations for the case $\mathfrak{J}_0 \geq 2$ .....	224
<b>References</b> .....	<b>233</b>
<b>Index</b> .....	<b>241</b>