

# Contents

<b>Introduction</b> .....	<b>1</b>
<b>Chapter 1. Preliminary Material: Cohomology, Currents</b> .....	<b>5</b>
1.A. Dolbeault Cohomology and Sheaf Cohomology .....	5
1.B. Plurisubharmonic Functions .....	6
1.C. Positive Currents .....	9
<b>Chapter 2. Lelong numbers and Intersection Theory</b> .....	<b>15</b>
2.A. Multiplication of Currents and Monge-Ampère Operators .....	15
2.B. Lelong Numbers .....	18
<b>Chapter 3. Hermitian Vector Bundles, Connections and Curvature</b> ..	<b>25</b>
<b>Chapter 4. Bochner Technique and Vanishing Theorems</b> .....	<b>31</b>
4.A. Laplace-Beltrami Operators and Hodge Theory .....	31
4.B. Serre Duality Theorem .....	32
4.C. Bochner-Kodaira-Nakano Identity on Kähler Manifolds .....	33
4.D. Vanishing Theorems .....	34
<b>Chapter 5. <math>L^2</math> Estimates and Existence Theorems</b> .....	<b>37</b>
5.A. Basic $L^2$ Existence Theorems .....	37
5.B. Multiplier Ideal Sheaves and Nadel Vanishing Theorem .....	39
<b>Chapter 6. Numerically Effective and Pseudo-effective Line Bundles</b> ..	<b>47</b>
6.A. Pseudo-effective Line Bundles and Metrics with Minimal Singularities .....	47
6.B. Nef Line Bundles .....	49
6.C. Description of the Positive Cones .....	51
6.D. The Kawamata-Viehweg Vanishing Theorem .....	56
6.E. A Uniform Global Generation Property due to Y.T. Siu .....	58
<b>Chapter 7. A Simple Algebraic Approach to Fujita's Conjecture</b> .....	<b>61</b>
<b>Chapter 8. Holomorphic Morse Inequalities</b> .....	<b>71</b>
8.A. General Analytic Statement on Compact Complex Manifolds .....	71
8.B. Algebraic Counterparts of the Holomorphic Morse Inequalities .....	72
8.C. Asymptotic Cohomology Groups .....	74
8.D. Transcendental Asymptotic Cohomology Functions .....	78
<b>Chapter 9. Effective Version of Matsusaka's Big Theorem</b> .....	<b>83</b>
<b>Chapter 10. Positivity Concepts for Vector Bundles</b> .....	<b>89</b>
<b>Chapter 11. Skoda's <math>L^2</math> Estimates for Surjective Bundle Morphisms</b> ..	<b>99</b>
11.A. Surjectivity and Division Theorems .....	99
11.B. Applications to Local Algebra: the Briançon-Skoda Theorem .....	105

<b>Chapter 12. The Ohsawa-Takegoshi <math>L^2</math> Extension Theorem</b> .....	<b>111</b>
12.A. The Basic a Priori Inequality .....	111
12.B. Abstract $L^2$ Existence Theorem for Solutions of $\bar{\partial}$ -Equations .....	112
12.C. The $L^2$ Extension Theorem .....	114
12.D. Skoda's Division Theorem for Ideals of Holomorphic Functions .....	122
<b>Chapter 13. Approximation of Closed Positive Currents by Analytic Cycles</b> .....	<b>127</b>
13.A. Approximation of Plurisubharmonic Functions Via Bergman kernels .....	127
13.B. Global Approximation of Closed (1,1)-Currents on a Compact Complex Manifold ...	129
13.C. Global Approximation by Divisors .....	136
13.D. Singularity Exponents and log Canonical Thresholds .....	143
13.E. Hodge Conjecture and approximation of $(p, p)$ - currents .....	148
<b>Chapter 14. Subadditivity of Multiplier Ideals and Fujita's Approximate Zariski Decomposition</b> .....	<b>153</b>
<b>Chapter 15. Hard Lefschetz Theorem with Multiplier Ideal Sheaves</b> .....	<b>159</b>
15.A. A Bundle Valued Hard Lefschetz Theorem .....	159
15.B. Equisingular Approximations of Quasi Plurisubharmonic Functions .....	160
15.C. A Bochner Type Inequality .....	166
15.D. Proof of Theorem 15.1 .....	168
15.E. A Counterexample .....	170
<b>Chapter 16. Invariance of Plurigenera of Projective Varieties</b> .....	<b>173</b>
<b>Chapter 17. Numerical Characterization of the Kähler Cone</b> .....	<b>177</b>
17.A. Positive Classes in Intermediate $(p, p)$ -bidegrees .....	177
17.B. Numerically Positive Classes of Type (1,1) .....	178
17.C. Deformations of Compact Kähler Manifolds .....	184
<b>Chapter 18. Structure of the Pseudo-effective Cone and Mobile Intersection Theory</b> .....	<b>189</b>
18.A. Classes of Mobile Curves and of Mobile $(n - 1, n - 1)$ -currents .....	189
18.B. Zariski Decomposition and Mobile Intersections .....	192
18.C. The Orthogonality Estimate .....	199
18.D. Dual of the Pseudo-effective Cone .....	202
18.E. A Volume Formula for Algebraic (1,1)-Classes on Projective Surfaces .....	205
<b>Chapter 19. Super-canonical Metrics and Abundance</b> .....	<b>209</b>
19.A. Construction of Super-canonical Metrics .....	209
19.B. Invariance of Plurigenera and Positivity of Curvature of Super-canonical Metrics ...	216
19.C. Tsuji's Strategy for Studying Abundance .....	217
<b>Chapter 20. Siu's Analytic Approach and Păun's Non Vanishing Theorem</b> .....	<b>219</b>
<b>References</b> .....	<b>223</b>