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Volume 16

# *Moduli Spaces and Locally Symmetric Spaces*

edited by  
Lizhen Ji and Shing-Tung Yau

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Moduli Spaces and Locally Symmetric Spaces

Edited by

Lizhen Ji (University of Michigan)

Shing-Tung Yau (Harvard University)

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# Preface

The notion of moduli spaces of Riemann surfaces was introduced by Bernhard Riemann in 1857 in his classical paper “Theory of abelian functions” and these moduli spaces and related spaces have been extensively studied since then. The most basic one is the moduli space of Riemann surfaces of genus 1, which turns out to be isomorphic to the most basic locally symmetric space, the quotient of the Poincaré upper half plane by the modular group, i.e., the moduli curve. Consequently, there is a lot of connection between moduli spaces and locally symmetric spaces since they are both generalizations of this special space.

This book consists of five expository papers on moduli spaces and locally symmetric spaces based on lecture notes given by the authors at two instructional workshops held at the Morningside Center of Mathematics, Beijing, in February 2017 and March 2019. They give accessible and systematic introductions to moduli spaces of Riemann surfaces, algebraic curves, moduli spaces of vector bundles on Riemann surfaces, moduli spaces of singularities, and compactification of a natural class of locally symmetric spaces. They should serve as good instructions to some important aspects of these important spaces.

We would like to thank the contributors of these papers for their efforts in writing up their lecture notes carefully and referees for their help. We also like to thank the staff members of the Morningside Center of Mathematics, especially Guangqiang Tie and Jing Long, for the kind help which made the workshops run smoothly.

*Editors: Lizhen Ji, Shing-Tung Yau  
April 2020*



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