# Proceedings of the Gökova Geometry-Topology Conferences 2018/2019

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## $25^{\mbox{th}}$ GÖKOVA GEOMETRY-TOPOLOGY CONFERENCE May 28 – June 2, 2018



Participants of the  $25^{\text{th}}$  Gökova Geometry-Topology Conference

## $26^{\mbox{th}}$ GÖKOVA GEOMETRY-TOPOLOGY CONFERENCE May 27 – June 1, 2019



Participants of the  $26^{\text{th}}$  Gökova Geometry-Topology Conference

#### PREFACE

This proceedings is dedicated to Michael Atiyah, it contains articles from both Gökova Geometry-Topology Conferences of 2018 and 2019 as well as independent contributions from S. Donaldson and C. Taubes–Y. Wu. We thank all the participants for making these conferences stimulating and enjoyable events, and thank to all the contributing authors. We thank NSF (National Science Foundation) and TMD (Turkish Mathematical Society) for funding these conferences. As always, we are grateful to Hotel Yücelen for supporting and hosting these conferences on the scenic shores of the Gökova Bay. Finally, we thank Üstün Yıldırım and Selahi Durusoy for formatting these proceedings.

September 2020

The editors

We dedicate this volume to the memory of Michael Atiyah, a great mathematician of  $20-21^{st}$  centuries. He is well known for his foundational work in Topology and Geometry, in particular in K-theory, Index theory, and Gauge theory. More recently he has done fundamental work on TQFT and physics. He was a supporter of Gökova Geometry Topology Conferences, which he attended 1996, and wrote an article on QFT in its 1997 proceedings. He had a very warm and friendly presence, always willing to give a friendly ear to younger mathematicians around him while giving good advice. My first personal encounter with him was in 1975, when I was a new PhD in IAS. I recall after leaving Marston Morse's party, we were walking on the institute grounds towards the housing together; he asked me "young man what are you working on?". I proceeded to tell him that I am working to surger the famous Kummer surface in  $\mathbb{CP}^3$  (by that time I had drawn so many handle-body pictures of that manifold, that I was convinced I am almost there), but to my disappointment he told me there should some obstructions doing that coming from spin structures! Many years later I kicked myself not heeding this prophetic remarks. Interacting with him was always a source of mathematical stimulation. I recall in the middle of the Gökova Conference dinner, where he was being honored with music and a candle lit cake, he suddenly remembered a young mathematicians earlier plea to hear her thesis, he went to a quiet corner of the room called her to listen her patiently. His Gökova visit left a lasting impression among the participants, especially in younger mathematicians who were in awe of him. Michael, this modest Gökova volume is to you from us, with heartfelt respect.

September 2020

Selman Akbulut

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