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Volume XXII

Surveys in Differential Geometry 2017

Celebrating the 50th Anniversary of the Journal of Differential Geometry

Lectures given at the Geometry and Topology Conference at Harvard University in 2017

editors

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Preface

In 1967, C.-C. Hsiung at Lehigh University had the vision to form a journal dedicated to geometry alone. Fifty years later, we celebrate the anniversary of the establishment of the *Journal of Differential Geometry*.

The subject of differential geometry has grown spectacularly since the *JDG* began publishing. Over the last fifty years, we have witnessed the revolution of geometry in several stages, including, for example: the work of McKean–Singer and Patodi which led to the proof of the local index formula; Milnor's relation of geometry to group theory, which in effect created the subject of geometric group theory; Bott's work on the residue formula for holomorphic vector fields; Freedman's solution to the four-dimensional Poincaré conjecture; Donaldson's use of the gauge theory to introduce Donaldson invariants, solving important problems in four-dimensional topology; Floer's important idea regarding the Arnold conjecture, which led to the Floer theory; Schoen and Uhlenbeck's regularity theory on harmonic maps; Hamilton's work on the Ricci flow, which led to spectacular developments in geometric flows and applications; Taubes' work on self-dual connections on 4-manifolds; and so on.

We refer the reader to the special five-volume set of *Selected Papers from the Journal of Differential Geometry, 1967–2017* for more highlights in the development of differential geometry over the past 50 years.

I took up the job of editing the *JDG* in 1980 when I was a faculty member at the Institute of Advanced Study. Hsiung called me several times to convince me to take up the job of editing. I initially refused the request because I did not have any experience as an editor, and because I was afraid the job may be too time-consuming. But after Calabi and Chern asked me, I found it difficult to refuse the duty of editing this important journal. Griffiths and Lawson also joined the team and played very important roles in the first few years of editing, but during that period the bulk of the work was carried out at the Institute for Advanced Study by their very capable secretaries, led by Miss Underwood. When I moved to the University of California at San Diego, and later to Harvard University, I again found capable secretaries, and I would like to thank all of these people, who worked hard for the well-being of the journal. A very important turning-point of the journal was its successful publication of the four-dimensional Poincaré

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conjecture due to Freedman, and the papers of Donaldson. The paper of Witten on Morse theory is equally important, as it has inspired many truly important works in mathematics and theoretical physics. I'd like to thank them all.

The *Journal of Differential Geometry* has also contributed to the mathematics community by hosting a major annual geometry and topology conference, held every third year at Harvard and the other two years at Lehigh University. The editors of this volume are thus indebted to the departments of mathematics at Harvard and at Lehigh, and we wish to take this occasion to express our deep gratitude to them.

At the 2017 *JDG* conference, we gathered a truly great group of geometers at Harvard University to celebrate the journal's 50th anniversary. Over thirty distinguished speakers presented their important works.

Several of those works appear in the present volume, and include: Denis Auroux on speculations on homological mirror symmetry for hypersurfaces in $\mathbb{C}^n$; Frances Kirwan on variation of non-reductive geometric invariant theory; Camillo De Lellis on the Onsager theorem; Simon Donaldson's remarks on G_2-manifolds with boundary; Daniel Freed on equivariant Chern–Weil forms and determinant lines; Kenji Fukaya on construction of Kuranishi structures on the moduli spaces of pseudo-holomorphic disks; Larry Guth on recent progress in quantitative topology; Blain Lawson on Lagrangian potential theory and a Lagrangian equation of Monge–Ampère type; Alena Pirutka on intersections of three-quadrics in $\mathbb{P}^7$; Bong Lian on period integrals and tautological systems; Yujiro Kawamata on birational geometry and derived categories; Fernando C. Marques on the space of cycles, a Weyl law for minimal hypersurfaces, and Morse index estimates; Duong Phong on new curvature flows in complex geometry; and Steve Zelditch on local and global analysis of nodal sets.

We wish to thank all the speakers at the 2017 conference, and especially those who contributed to this special volume of the *Surveys in Differential Geometry* book series.

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