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Complex Geometry from Riemann to Kähler–Einstein and Calabi–Yau

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Advanced Lectures in Mathematics, Volume 38
Complex Geometry from Riemann to Kähler–Einstein and Calabi–Yau

Volume Editor:
Lizhen Ji (University of Michigan, Ann Arbor)

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Preface

Complex geometry has been extensively studied and developed since the 19th century. It is now one of the central subjects in mathematics. History has played a more important role in mathematics than in other disciplines. The reason is that whatever was proved in mathematics will remain true forever, and hence there is a direct continuity of the past, the present and the future of mathematics. On the other hand, in the huge literature of mathematics, only a few works stand out and exert a big influence on the development of mathematics. A book consisting of seminal papers in complex geometry together with some commentaries will be a valuable source for both students and experts, who want to learn and appreciate a global and historical picture of the subject.

Since Bernhard Riemann is one of the originators of complex geometry and differential geometry, this book includes a selection of the thesis of Riemann on the foundation of complex analysis and his masterpiece on the foundation of geometry. It also includes papers of modern masters: Atiyah, Bott, Chern, Calabi, Chow, Donaldson, Hirzebruch, Kähler, Kodaira, Siu, Uhlenbeck, and Yau. To motivate the selection of these papers, a brief history of complex geometry based on extensive quotes from many experts on both mathematics and research in mathematics is included at the beginning of this book. At the beginning of each selected paper, we have included a brief explanation for the selection.

To see the development of contemporary mathematics through the eyes of one of the most active practitioners and leaders, the book contains a set of commentaries written by Prof. Shing-Tung Yau from his personal perspective on the broad subject of geometric analysis, in particular complex geometry, and its many applications, especially to mathematics physics.

Since only a limited number of papers are reprinted in this book, we have also compiled a list of important papers in geometric analysis in the 20th century. This list is the fruit of many intense and extensive email correspondences with Prof. Shing-Tung Yau and should reflect the perspective of Prof. Yau on the broad subject of geometric analysis. It appears as the last chapter and, we hope, should serve as a guide to geometric analysis and some related subjects.

Lizhen Ji
August 2016

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