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Hodge Theory and L^2-analysis

edited by

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Advanced Lectures in Mathematics, Volume 39
Hodge Theory and L^2-analysis

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ADVANCED LECTURES IN MATHEMATICS

Dedicated to Steven Zucker for his 65th Birthday

Conference on Hodge Theory and L^2-cohomology, Johns Hopkins University, November 21—23, 2014

Contents

Preface

Steven (Steve) Zucker has made major contributions to the following three areas of mathematics and the interactions among them:

1. Hodge theory in algebraic geometry (e.g., normal functions, variation of mixed Hodge structure, degeneration of Hodge structure);

2. L^2-cohomology, and also L^p-cohomology for $p \neq 2$;

3. Compactification of locally symmetric spaces (Satake, reductive Borel-Serre, toroidal, etc.).

The famous Zucker conjecture from 1980 relates items (2) and (3) above. It was resolved independently in 1987 by Saper-Stern and by Looijenga, using very different methods. For a broader, detailed description of Steve's work, see his own narrative which follows.

In November 2014, a conference titled "Hodge Theory and L^2-cohomology" was held in Steve's honor at Johns Hopkins University. This book is based on the conference. Besides contributions from most of the speakers, several other people were also invited to contribute. Since Steve's work and some of the contributed papers involve L^2-analysis, which includes L^2-cohomology, the title of the book was changed to "Hodge Theory and L^2-analysis". By consensus, the conference was of high quality. We believe that this book reflects the conference well and is also of high quality.

It perhaps helpful to emphasize the intimate connection between Hodge theory and L^2-analysis. In his thesis in 1851, Riemann tried to use the Dirichlet form and the Dirichlet principle to prove the Riemann mapping theorem. In his very influential paper on abelian functions in 1857, he also used harmonic functions and the Dirichlet principle to prove the Riemann inequality in the Riemann-Roch theorem. These results were made rigorous and systematically presented in Weyl's classical book *The Concept of a Riemann Surface* which was published in 1913. Some of the results for Riemann surfaces were later generalized to higher dimension Riemannian and Kähler manifolds by Hodge, Weyl and Kodaira, whose works gave birth to the Hodge theory.

The contributors to this book had been asked to make their papers expository to the extent possible. We hope that this book will serve as a valuable introduction to Hodge theory, L^2-analysis and the interaction between the two. All papers have been carefully refereed, and we would like to thank the referees for their help.

The conference was held at approximately the 65th birthday of Steve. We wish him many happy and productive years to come.

Lizhen Ji

September, 2016

The Research Career of Steven Zucker: An Autobiographical Account

Acknowledgments. I thank Rafe Mazzeo, Arvind Nair, and Morihiko Saito for their constructive comments on earlier versions of this narrative. I must thank in particular Benjamin Diamond, who served as the copy editor of its penultimate version.

For the reader's convenience, I divide the references to my work to date among the three areas mentioned in the Preface.

(1) Hodge theory in algebraic geometry (normal functions, variation of mixed Hodge structure, degeneration of Hodge structure, etc.): [Z01]–[Z06], [Z09], [Z12]–[Z23];

(2) L^2-cohomology, and also L^p-cohomology for $p \neq 2$: [Z03], [Z07], [Z10], [Z24], [Z28];

(3) Compactification of locally symmetric spaces (Satake, reductive Borel-Serre, toroidal, etc.): [Z08], [Z20]–[Z22], [Z24]–[Z28].

In what follows, there will be no false modesty, and I will avoid caustic remarks. In addition to a description of my work, I will acknowledge the major influences of other mathematicians, as well as including the impact of my own work on that of others. I wish to put in the records that I gave general (moral) support to two first-rate mathematicians who were getting a bit discouraged while in graduate school: Kari Vilonen and Teruyoshi Yoshida. I know now that Yoshida remembers this well and with appreciation; Vilonen is less clear, but he believes it is accurate. I mention this to remind the reader that there are ways of helping a novice beyond the imparting of mathematical ideas.

I was born in New York City on September 12, 1949. A year later, my parents (and I) moved to Queens Village, located in the Borough of Queens, NYC, where my father had set up his optometric practice. I went to public elementary, junior high school (a.k.a. middle school) and high school. All were in walking distance from our home, but we lived even closer to the city line. That precluded the idea of going to a selective high school, for the commute would have been way too long; besides, the local high school was rather good. I have one sibling, a brother, born in 1952.