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# Nonlinear Analysis in Geometry and Applied Mathematics

Editors
Lydia Bieri
Piotr Chruściel
Shing-Tung Yau

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Nonlinear Analysis in Geometry and Applied Mathematics

Editors:

Lydia Bieri
University of Michigan

Piotr T. Chruściel
University of Vienna and the Erwin Schrödinger Institute

Shing-Tung Yau, *editor-in-chief*
Director of the CMSA, Harvard University



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# Preface

Nonlinear partial differential equations (PDEs) on the one hand describe phenomena in science, technology, and related fields; and on the other hand provide an excellent excuse for fascinating mathematical research. Nonlinear PDEs comprise various subfields concerned with different phenomena. Within this larger frame we find *mathematical general relativity*, with Einstein equations at its core. The latter are a system of non-linear (in fact quasi-linear) PDEs that can be brought into hyperbolic form. They describe our universe by relating its geometric structures to matter and energy content. Hereby, gravitation acts through curvature. Solutions of the Einstein equations are Lorentzian metrics, and the universe or its parts are modeled as manifolds equipped with these metrics; we call these spacetimes. Whenever other fields are present, they obey their own equations and we obtain a coupled system. Examples include the Einstein–Maxwell and the Einstein–Euler systems.

The main goal of mathematical general relativity is to understand the global structure of solutions of Einstein equations. This is often done by solving the Cauchy problem for the Einstein equations—with initial data corresponding to situations of physical interest—and studying the resulting spacetimes. Mathematical general relativity has thus solved challenging mathematical problems and given answers to long-standing questions in physics. Along the way, it has led to many intriguing new questions in mathematics, pushing further research in related areas.

During the 2015–2016 year at the Harvard Center of Mathematical Sciences and Applications (CMSA), several researchers working in mathematical general relativity presented lectures on modern topics of research in the field of "Non-linear Equations." This volume presents articles—by those researchers and their co-authors—drawn from those CMSA lectures.

Specific topics include the Cauchy problem for the Einstein equations in cosmological and non-cosmological settings; investigation of stability as well as singularities (black holes) of classes of spacetimes; initial data engineering; gravitational radiation; and asymptotics of spacetimes, quasi-local energies, and their limits.

The content of this volume reflects some of the activities at the Harvard CMSA during the 2015–2016 program, and provides insights into active areas of research in mathematical general relativity that can benefit scholars working in PDEs, geometric analysis, and general relativity.

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The Editors:
Lydia Bieri
Piotr T. Chruściel
Shing-Tung Yau