

Advanced Lectures in Mathematics
Volume 35.2

The Legacy of Bernhard Riemann After One Hundred And Fifty Years

Volume II

Companion to the volume

The Legacy of Bernhard Riemann After One Hundred and Fifty Years, Volume I

edited by

Lizhen Ji · Frans Oort · Shing-Tung Yau

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The Legacy of Bernhard Riemann After One Hundred and Fifty Years, Volume I

Editors:
Lizhen Ji (University of Michigan)
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