

Advanced Lectures in Mathematics
Volume 35.1

The Legacy of Bernhard Riemann After One Hundred And Fifty Years

Volume I

Companion to the volume

The Legacy of Bernhard Riemann After One Hundred and Fifty Years, Volume II

edited by

Lizhen Ji · Frans Oort · Shing-Tung Yau

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Editors:

Lizhen Ji (University of Michigan)

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