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MORNINGSIDE LECTURES IN MATHEMATICS
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Lectures on the Analysis of Nonlinear Partial Differential Equations: Part 4

edited by

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Morningside Lectures in Mathematics, Volume IV
Lectures on the Analysis of Nonlinear Partial Differential Equations: Part 4

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Preface to the First Volume

In summer of 2001, we initiated a summer school program on the harmonic analysis and its applications in nonlinear partial differential equations, with special emphases on nonlinear Schrödinger equations, kinetic equations of Boltzmann type and classical fluid equations. Over the years, there have been many distinguished mathematicians working in these fields who have come to help our program and to give series of special lectures. The program has been shown to be particularly helpful to young researchers and students. The lectures involved have gradually turned into more formal and regular seminars on *Analysis in Partial Differential Equations* at the Morningside Center of Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences.

From June 2007 to January 2008, we held a special semester PDE-program. We invited many mathematicians and experts in mathematical theory of fluid mechanics and quantum mechanics. The visitors during that period include: Bresch Didier, Carles Rémi, Jean-Yves Chemin, Desvillettes Laurent, Lopes Filho Milton C, Nussenzweig Lopes Helena J., Novotny Antonin, Chao-Jiang Xu, Chongchun Zeng, Ping Zhang and Yuxi Zheng, who gave a series of lectures and provided excellent lecture notes. It is no doubt that these lecture notes would be very useful for many researchers and students. In this volume, we have collected lecture notes by M. C. Lopes concerning the boundary layers of incompressible fluid flow; by C. J. Xu on the micro-local analysis and its applications to the regularities of kinetic equations; by Y. X. Zheng on the weak solutions of variational wave equation from liquid crystals; and by P. Zhang and Z. F. Zhang on the free boundary problem of Euler equations. In addition, we also included the notes by F. Nier on the hypoellipticity of Fokker-Planck operator and Witten-Laplace operator that were given earlier in the summer of 2006.

We have planned to publish in the forthcoming volumes the other lecture notes. Some are from past lectures at our program and some will be collected from the newly scheduled seminars. We hope that the publication of these lecture notes may provide valuable references and up-to-date descriptions of current developments of various and related research topics, that will benefit many young

researchers or graduate students. We wish to take this opportunity to thank the Morningside Center of Mathematics, the Institute of Mathematics of AMSS that provides all necessary supports. We are particularly grateful to professor Lo Yang for his constant help, supports and encouragements to our program. We also would like to thank Guilong Gui for his careful preparations of the Latex file of the entire book. We finally appreciate for the financial support from the Chinese Academy of Sciences.

Fanghua Lin in New York

Xueping Wang in Nantes

Ping Zhang in Beijing

November 3, 2008

Preface to the Fourth Volume

This book is a sequel to the previous volumes *Lectures on the Analysis of Nonlinear Partial Differential Equations*, Vol. I, Vol. II, and Vol. III. In this fourth volume we have collected those lecture notes by Jean-Yves Chemin, Hongjie Dong, Xiaochun Li, Fanghua Lin, Alexis F. Vasseur, Jiahong Wu, and Xiaoyi Zhang. We appreciate very much their time and efforts to provide us with these excellent notes. We believe these lecture notes will serve as valuable references on current developments in various research topics in nonlinear partial differential equations (PDE).

The notes collected here came initially from the seminars on Analysis in Partial Differential Equations. It was a part of the PDE program (from April through October 2012) at the Academy of Mathematics and Systems Science, Chinese Academy of Sciences (CAS). We would like to take this opportunity to thank the National Center for Mathematics and Interdisciplinary Sciences, Academy of Mathematics and Systems Science as well as the Morningside Center of Mathematics of CAS for the financial support. We thank also several other distinguished visitors who had delivered excellent lectures at the program: Hammadi Abidi, Albert Cohen, Marius Paicu, Benoit Perthame and Jean-Claude Saut. We wish their notes would be included in our future volumes.

Finally, we wish to thank Prof. Lo Yang, Prof. Nanhua Xi, and Prof. Yuefei Wang, for their continuing support over all these years which was essential to the success of this PDE program. We also thank Dr. Guilong Gui for the careful preparation of the Latex files that make up this entire book.

Jean-Yves Chemin in Paris

Fanghua Lin in New York

Ping Zhang in Beijing

June, 2015

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