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Volume 34

# Some Topics in Harmonic Analysis and Applications

edited by

Junfeng Li · Xiaochun Li · Guozhen Lu

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Some Topics in Harmonic Analysis and Applications

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*This special volume is dedicated to  
Professor Shanzhen Lu*





# Preface

This is a special volume dedicated to Professor Shanzhen Lu on the occasion of his 75th birthday. As friends and students of Professor Lu, it is our great pleasure to edit this volume to celebrate the mathematical life and career of Professor Lu.

Professor Lu has made important mathematical contributions in harmonic analysis and applications. In addition, Professor Lu has educated and trained many younger mathematicians in China. Professor Lu supervised around 50 M.S. and Ph.D. students, post-doctors and visiting scholars over the years. We refer the reader to “A List of Ph.D. Students, Post-doctors and Visiting Scholars Supervised by Professor Shanzhen Lu and Foreign Collaborators Who Worked with Professor Shanzhen Lu”. We also refer the reader to “An Introduction of Professor Shanzhen Lu” for more information about his personal and mathematical life.

This volume describes some recent development on some topics in harmonic analysis and applications to partial differential equations and geometric analysis. Experts in these areas are invited to contribute works which address recent development in some of the topics. Subjects covered in this volume include local  $Tb$  theorems in non-homogeneous or vector-valued setting, multilinear embedding theorems, norm estimates of quasi-modes on Riemannian manifolds, Calderón-Zygmund singular integral operators and commutators, Dirichlet-to-Neumann maps, multilinear and multi-parameter Fourier multipliers, weighted normed inequalities, nonlinear Schrödinger equations, function space theory, etc. It will be an excellent reference book for mathematicians working in analysis and PDEs and advanced graduate students, post-doctors in these and related areas.

We are indebted to many people who have contributed to the publication of this volume. First of all, we would like to take this opportunity to sincerely thank all contributors of articles to this volume. This volume could not have been completed without their hard work and mathematical contributions. In particular, we like to thank them for their patience in waiting for this volume to be finally published. We also like to thank Guido Weiss, Elinor Anheuser Professor of Mathematics at Washington University in St. Louis, for his short tribute to Professor Shanzhen Lu. Professor Lu spent two years at Washington University working with Professors Weiss and Mitchell Taibleson in early 1980's. We also thank Editors of the series of Advanced Lectures in Mathematics Professors S.-T. Yau, K. Liu, L. Ji for their support in the process of editing of this volume. In particular Lizhen has always been a good person to talk to when we have questions. Most importantly, special

thanks are due to Ms. Huaying Li and Ms. Liping Wang from the Higher Education Press in China for their tireless support and hard work in editing this volume. Without the tremendous support and devotion of Ms. Huaying Li and Ms. Liping Wang and many other staff members at the Higher Education Press, we could not have had completed this well organized volume. In particular, this is the second time for the third Editor (Guozhen Lu) of this special volume to collaborate with both of Ms. Huaying Li and Ms. Liping Wang. It is indeed a pleasure for him to work with them through this long process.

Lastly, it is our great pleasure and honor to be in charge of editing this volume in celebration of the mathematical life of Professor Lu. All of us have benefited from Professor Lu in many aspects over the years. As former students and/or friends of Professor Lu, we would like to take this opportunity to thank Professor Lu for his teaching, continuous support to our mathematical careers and his long-lasting friendship.

Junfeng Li, Beijing Normal University, China  
Xiaochun Li, University of Illinois at Urbana-Champaign, USA  
Guozhen Lu, Wayne State University, USA  
May, 2015

# A Tribute to Professor Shanzhen Lu

Professor Shanzhen Lu is a dear friend who has contributed considerably to the mathematical world. I first met Professor Lu when he came to the Department of Mathematics at Washington University in St. Louis to work with me in the fall of 1980. Professor Lu remained in St. Louis until the end of the spring semester in 1982. During that time we developed a wonderful friendship and collaboration. This friendship continues to this day in the year 2015.

I am proud to say that I have many close Chinese friends as well as having had some excellent Chinese Ph.D. students. These rich connections have permitted me to visit China often and I always made times to see Professor Lu and reestablish our old ties. Professor Lu and all of my other substantial relationships with Chinese mathematicians have helped make my career a wonderful experience.

Professor Lu and I began collaborating in harmonic analysis soon after his arrival in St. Louis. It is important to state that the late Professor Mitchell Taibleson joined this collaboration. On several occasions Professor Taibleson expressed his pleasure with being a part of this joint work.

Professor Lu and I studied and developed many properties of important operators in harmonic analysis in the study of certain function spaces that we described as being generated by blocks. “Blocks” are generalizations of “atoms”. The latter is an important notion in the study and extension of Hardy spaces. In 1989, Professor Lu wrote a book on the theory of spaces generated by blocks. He very generously included my name and that of Professor Taibleson as co-authors of this book.

In this short note I cannot describe in great detail Professor Lu’s research. However, it is quite extensive and deals with many topics in analysis. Let me name some of the areas in which he has worked: Hardy spaces and their extensions to higher dimensions, applications of Fourier integrals, almost everywhere convergence of important means of multiple Fourier series and the boundedness of several important operators.

Professor Lu was born in November 1939. In 1961 he graduated from East China Normal University. In 1961–1980 he was an Assistant Professor in Beijing Normal University and was promoted in 1983 to a Full Professor in this institution. At that time, Professor Lu was the Director of the Institute of Mathematical

Sciences and Education. He was Chancellor of Beijing Normal University during the period 1995–1999. He also was a member of the Editorial Board of *Acta Mathematica Sinica* and the *International Journal of Applied Mathematical Sciences*. In addition, he has received a number of important prizes in both education and research.

From this short history it is clear that Professor Lu has had a most distinguished, varied and impressive career. I am very proud to have been his friend and collaborator over these many years.

Guido Weiss  
Washington University in St. Louis  
February 19, 2015

# An Introduction of Professor Shanzhen Lu

Professor Shanzhen Lu was born in 1939 in Wenzhou, a coastal city in the east of China. Lu was the second of an eight-children family. It was very tough to take care of such a big family only with the salary of an office worker during the war time. But his parents still tried their best to let him finish his high school in 1957. Wenzhou, now a famous city by its rapid commercial development, is also thought to be the hometown of mathematicians in China. Many renowned mathematicians in modern China call Wenzhou as their hometown. As one of the early opened cities in China after the Opium War in 1840, its education in science was better than most other parts of China at that time, especially in the mathematical education. Some excellent mathematicians were teaching in high schools those years. Lu was lucky to have the opportunity to access to some advanced mathematics starting his years during high school study. The beauty of mathematics attracted this young man. He made up his mind to study mathematics further. From 1957 to 1961, he studied at the Department of Mathematics of East China Normal University. He graduated in 1961 and moved to Beijing. From that on, he has worked at Beijing Normal University for more than 50 years as a faculty member.

As an outstanding mathematician who specializes in harmonic analysis, Professor Shanzhen Lu has made significant contributions in a number of subjects in harmonic analysis and their applications. His works involve many important fields, such as the boundedness of the Bochner-Riesz means operators, which was motivated to set up the pointwise convergence of Fourier series in high dimension spaces. He has also done important works on the boundedness of singular integral operators with rough kernels, the oscillatory integrals and their commutators. Function space theory is also one of his research interests. He obtained many interesting results in the theory of Hardy spaces and Herz type spaces.

Professor Lu is one of the pioneers helping the development of modern harmonic analysis in China. Together with a few others in the older generation, he introduced the most updated development of the real variable method on Euclidean spaces, an important branch of harmonic analysis, to the mathematical community in China. In the beginning of 1980's, the research of harmonic analysis in China was very weak. After Lu and his colleague's forty-year long effort and contribution, most of major universities in China now have faculty members working in this important area. Professor Lu himself has also trained many students

and visiting scholars. Among 29 people who obtained their Ph.D under Professor Lu's supervision, most of them now are active harmonic analysts and are leading members of faculty in their own universities in China.

In addition to his achievement in mathematics and education, he is also a successful administrator. In 1994, he became the first elected president of Beijing Normal University, a top research university focusing on liberal arts and sciences in China. He was also one of the first two elected presidents of Chinese universities since the founding of the People's Republic of China. As a president of a university in China, he realized that the most important thing is the quality of the faculty, especially the healthy growth of young faculty members. He paid a considerable attention to support young faculty members by implementing some new promotion policy that was designed for helping young faculty members to improve their research and teaching quality and to grow up to a leader as soon as possible. More than 15 years have passed since he was the president of Beijing Normal University, Professor Lu has been making substantial contributions to education in China. He was invited in 2012 to become the founding president of Wenzhou-Kean University, a joint adventure between Kean University in US and municipal government of the city of Wenzhou.

Professor Lu is a passionate person who loves Chinese 5000-year history and culture. He is a great teacher holding Chinese traditional wisdom and modern visions. All of his students have benefited from and been influenced by those precious characters through his teaching.

At his leisure time, Professor Lu enjoys the classical music and traditional Chinese opera. He is also good at swimming and he swam almost everyday when he was younger.

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