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ALM 33: Introduction to Modern Mathematics

Advanced Lectures in Mathematics Volume 33

# Introduction to Modern Mathematics

edited by

Shiu-Yuen Cheng · Lizhen Ji · Yat-Sun Poon

Jie Xiao · Lo Yang · Shing-Tung Yau





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