

Surveys in Differential Geometry

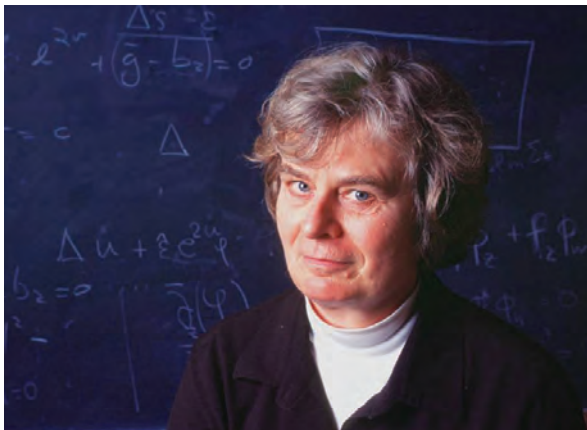
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Richard Hamilton



Leon Simon



Karen Uhlenbeck

Volume XIX

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2014

Regularity and evolution of
nonlinear equations

*Essays dedicated to Richard Hamilton, Leon Simon,
and Karen Uhlenbeck*

edited by

Huai-Dong Cao
Richard Schoen
Shing-Tung Yau



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Essays dedicated to Richard Hamilton, Leon Simon, and Karen Uhlenbeck

Volume Editors:

Huai-Dong Cao (Lehigh University)

Richard Schoen (Stanford University)

Shing-Tung Yau (Harvard University)

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Preface

Surveys in Differential Geometry 2014 is dedicated to the three most eminent contributors to the subject of regularity and existence of nonlinear partial differential equations, which has played such an important role in geometry. These are Richard Hamilton, Leon Simon, and Karen Uhlenbeck—all of whom have turned, or will turn, 70 within a year or so.

This special “birthday” volume, therefore, consists of articles written by their friends and former students. Topics include: analysis related to minimal submanifolds, Yang-Mills theory, Kähler metrics, Monge-Ampère equations, curve flows, and general relativity.

The editors would like to express our thanks and gratitude to all those who have contributed articles to this volume, and our congratulations to professors Hamilton, Simon, and Uhlenbeck.

Huai-Dong Cao
Lehigh University

Richard Schoen
Stanford University

Shing-Tung Yau
Harvard University

Contents

<i>Preface</i>	v
Moduli of continuity, isoperimetric profiles, and multi-point estimates in geometric heat equations BEN ANDREWS	1
Rigidity and minimizing properties of quasi-local mass PO-NING CHEN AND MU-TAO WANG	49
Lojasiewicz inequalities and applications TOBIAS HOLCK COLDING AND WILLIAM P. MINICOZZI II	63
The regularity of solutions in degenerate geometric problems PANAGIOTA DASKALOPOULOS	83
Algebraic families of constant scalar curvature Kähler metrics SIMON DONALDSON	111
Hermitian-Yang-Mills connections and beyond JUN LI	139
Interior a priori estimates for the Monge-Ampère equation JIAKUN LIU AND XU-JIA WANG	151
Dispersive geometric curve flows CHUU-LIAN TERNG	179
Regularity of stable minimal hypersurfaces: recent advances in the theory and applications NESHAN WICKRAMASEKERA	231