Handbook of Nonconvex Analysis and Applications

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edited by
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Preface

Nonconvex analysis is a rapidly developing, multi-disciplinary field of research, comprehending theoretical analysis in mathematical modelling of natural systems, bifurcation and chaos in dynamical systems, finite deformation theory, nonlinear partial differential equations, global optimization, calculus of variation, numerical methods, and scientific computations. The field of nonconvex analysis has undergone considerable development in a remarkably short time – with extensive applications to theoretical physics, material science, modern mechanics, complex systems, and scientific computations.

The present volume, *Handbook of Nonconvex Analysis and Applications,* was proposed by Shing-Tung Yau, a world-renowned mathematician at Harvard University, and we are honored to have been invited to edit it. The *Handbook* will serve as a much-needed reference work for the dynamic and ever-growing field of nonconvex analysis and its applications.

The *Handbook* consists of thirteen chapters written by notable experts in the field, addressing essential recent developments in nonconvex analysis and its applications, and keeping a balance between major areas of theory, methods, and applications. Each chapter provides an illuminating exposition of state-of-the-art approaches to a specific topic, with discussions of the central contributions, and pointers to some basic references. A variety of topics regarding nonconvex analysis and its applications are discussed: nonconvex variational principles; comparison principles; nonlinear eigenvalue problems; critical point theory; boundary value problems; topological methods, including Morse theory; nonlinear elliptic equations; evolution problems; difference equations; inequality problems; geometric properties of functions and spaces; and applications in mechanics.

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David Gao
Dumitru Motreanu

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