Surveys of Modern Mathematics
Volume VIII

Lie-Bäcklund-Darboux Transformations

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International Press
www.intlpress.com

高等教育出版社
HIGHER EDUCATION PRESS

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ISBN: 978-1-57146-288-6

Printed in the United States of America.

18 17 16 15 14   1 2 3 4 5 6 7 8 9

SURVEYS OF MODERN MATHEMATICS

Preface

One of the mathematical miracles of the 20th century was the discovery of a group of nonlinear wave equations being integrable. These integrable systems are the infinite dimensional counterpart of the finite dimensional integrable Hamiltonian systems of classical mechanics. Icons of integrable systems are the KdV equation, sine-Gordon equation, nonlinear Schrödinger equation etc. The beauty of the integrable theory is reflected by the explicit formulas of nontrivial solutions to the integrable systems. These explicit solutions bear the iconic names of soliton, multi-soliton, breather, quasi-periodic orbit, homoclinic orbit (the focus of this book) etc. There are several ways now available for obtaining these explicit solutions: Bäcklund transformation, Darboux transformation, and inverse scattering transform. The clear connection among these transforms is still an open question although they are certainly closely related. These transformations can be regarded as the counterpart of the canonical transformation of the finite dimensional integrable Hamiltonian system. Bäcklund transformation originated from a quest for Lie's second type invariant transformation rather than his tangent transformation. That brings the title of this book: Lie-Bäcklund-Darboux Transformations which refer to both Bäcklund transformations and Darboux transformations.

The most famous mathematical miracle of the 20th century was probably the discovery of chaos. When the finite dimensional integrable Hamiltonian systems are under perturbations, their regular solutions can turn into chaotic solutions. For such near integrable systems, existence of chaos can sometimes be proved mathematically rigorously. Following the same spirit, one may attempt to prove the existence of chaos for near integrable nonlinear wave equations viewed as near integrable Hamiltonian partial differential equations. This has been accomplished as summarized in the book [69]. The key ingredients in this theory of chaos in partial differential equations are the explicit formulas for the homoclinic orbit and Melnikov integral. The first author's taste is to use Darboux transformation to obtain the homoclinic orbit and Melnikov integral. This will be the focus of the first part of this book.

The second author's taste is to use Darboux transformation in a diversity of applications especially in higher spatial dimensions. The range of applications crosses many different fields of physics. This will be the focus of the second part of

this book. This book is a result of the second author's several visits at University of Missouri as a Miller scholar.

The first author would like to thank his wife Sherry and his son Brandon, and the second author would like to thank his wife Alla and his son Valerian, for their loving support during this work.

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About the Authors

Y. Charles Li works in the areas of chaos in partial differential equations, Navier-Stokes equations and turbulence, nano-technology, biological mathematics, and complex systems. His honors include a Guggenheim Fellowship, an AMS Centennial Fellowship, and the Princeton University Merit Prize. His published books include *Invariant Manifolds and Fibrations for Perturbed Nonlinear Schrödinger Equations, volume 128 of the Applied Mathematical Sciences series* (Springer-Verlag, 1997), and *Chaos in Partial Differential Equations* (International Press of Boston, 2004).

Artyom Yurov works in the area of mathematical physics, with expertise in applying Darboux transformations to various physical problems.