

Surveys of Modern Mathematics
Volume VII

Introductory Lectures on Manifold Topology: Signposts

Thomas Farrell

*Department of Mathematical Sciences
Binghamton University*

Yang Su

*Academy of Mathematics and Systems Science
Chinese Academy of Sciences*



International Press
www.intlpress.com



高等教育出版社
HIGHER EDUCATION PRESS

Surveys of Modern Mathematics, Volume VII
Introductory Lectures on Manifold Topology: Signposts

Thomas Farrell

Department of Mathematical Sciences, Binghamton University

Yang Su

Academy of Mathematics and Systems Science, Chinese Academy of Sciences

Copyright © 2014 by International Press, Somerville, Massachusetts, U.S.A., and by
Higher Education Press, Beijing, China.

This work is published and sold in China exclusively by Higher Education Press
of China.

All rights reserved. Individual readers of this publication, and non-profit libraries acting
for them, are permitted to make fair use of the material, such as to copy a chapter for use
in teaching or research. Permission is granted to quote brief passages from this
publication in reviews, provided the customary acknowledgement of the source is given.
Republication, systematic copying, or mass reproduction of any material in this pub-
lication is permitted only under license from International Press. Excluded from these
provisions is material in articles to which the author holds the copyright. (If the author
holds copyright, notice of this will be given with the article.) In such cases, requests for
permission to use or reprint should be addressed directly to the author.

ISBN: 978-1-57146-287-9

Printed in the United States of America.

18 17 16 15 14 1 2 3 4 5 6 7 8 9

SURVEYS OF MODERN MATHEMATICS

Series Editors

Shing-Tung Yau
Harvard University
Cambridge, Massachusetts
U.S.A.

Lizhen Ji
University of Michigan, Ann Arbor
U.S.A.

Yat-Sun Poon
University of California at Riverside
U.S.A.

Jie Xiao
Tsinghua University
Beijing, China

Jean-Pierre Demailly
Institut Fourier
Laboratoire de Mathématiques
Saint-Martin d'Hères, France

Eduard J. N. Looijenga
Mathematisch Instituut
Universiteit Utrecht
The Netherlands

Neil Trudinger
Mathematical Sciences Institute
Australian National University
Canberra, Australia

Dedicated to my mother with deepest gratitude

– Y. S.

Preface

The purpose of this book is to introduce to advanced graduate students and other interested mathematicians some of the basic technique and results from manifold topology. It is assumed that the reader is familiar with algebraic topology through cup products and Poincaré duality as well as with fiber bundles and characteristic classes; e.g. with the material in the first half of the book “Characteristic Classes” by J. W. Milnor and J. D. Stasheff. A glance at the Contents shows the topics that are covered. The book is based on a course of lectures given by the first author during the fall semester, 2009 at the Morningside Center of the Chinese Academy of Sciences. It was originally planned as a year long course; hence some of the topics alluded to in the Introduction are not covered here. These will be done in a second volume.

The writing of this book was partially supported by a grant from the National Science Foundation of the USA and by a Visiting Professorship at the Chinese Academy of Sciences of the first author, and by a grant from the National Science Foundation of China of the second author.

Contents

1	Introduction	1
2	The h-Cobordism Theorem	5
2.1	The h -Cobordism Theorem and Generalized Poincaré Conjecture ..	5
2.2	Tangent vectors, embeddings, isotopies	9
2.3	Handles and handlebody decomposition	13
2.4	Calculus of handle moves	18
2.5	Proof of the h -Cobordism Theorem	29
3	The s-Cobordism Theorem	35
3.1	Statement of the s -Cobordism Theorem	35
3.2	Whitehead group	40
3.3	Whitehead torsion for chain complexes	43
4	Some Classical Results	53
4.1	Novikov's Theorem	53
4.2	A counterexample to the Hurewicz Conjecture	55
4.3	Milnor's exotic spheres	58
4.4	Rochlin's Theorem	61
4.5	Proof of Novikov's Theorem	64
4.6	Novikov Conjecture	72
5	Exotic Spheres and Surgery	75
5.1	Plumbing	75
5.2	Surgery	80
6	Hauptvermutung	89
6.1	The Fundamental Theorem of algebraic K -theory	89
6.2	Edwards-Cannon's example	98
6.3	The Hauptvermutung	102
6.4	Whitehead torsion	103
6.5	Proof of Stallings' Theorem	108
6.6	Farrell-Hsiang's example	112

6.7	The structure set	115
6.8	Siebenmann's example	120
	References	125
	Index	127

Index

E_8 Dynkin diagram, 78
 K -flat, 115
 $K_0(R)$, 90
 $K_1(R)$, 40
 $Nil(R)$, 89
 $SK_1(R)$, 41
 Θ_n , group of homotopy spheres, 78
 $\overline{K}_1(R)$, 44
 bP_{n+1} , 78, 83
 h -Cobordism Theorem, 7
 h -cobordism, 6
 h -cobordism torsion, 105
 duality theorem, 107
 s -Cobordism Theorem, 39

A

algebraic cancellation, 26
almost parallelizable, 61
anchor ring, 65
Arf invariant, 81
attaching disc, 19
attaching sphere, 19
augmentation, 36, 44

B

Bass projection, 92
Bass-Heller-Swan formula, 112
Bernoulli number, 67, 83
Borel Conjecture, 73
Bott Periodicity Theorem, 56

C

clutching construction, 56
cobordism, 6
combinatorial structure on S^n , 102

complementary disc, 19
complementary sphere, 19

E

Edwards-Cannon Theorem, 101
embedding, 11
engulfing, 114
exotic sphere, 78, 83
 Milnor's, 59

F

fibering theorem, 65
Fundamental Theorem of algebraic
 K -theory, 89

G

Generalized Poincaré Conjecture, 5
geometric cancellation, 23
Grothendieck construction, 92

H

handle, 13
 index of, 13
handle addition, 26
handlebody decomposition, 14
 $(2, 3)$ -, 35
 dual, 22
 regular, 29
 standard, 20
 t-standard, 21
Hauptvermutung, 94, 102
Higman's trick, 95
Hirzebruch Index Theorem, 60, 67, 72
homology sphere, 100
 Poincaré homology sphere, 100

homotopy sphere, 78
 Hurewicz Conjecture, 3, 55

I

immersion, 11
 incidence matrix, 21
 integral group ring valued, 37
 integral group ring, 36
 isotopy, 11
 Isotopy Extension Theorem, 11

K

Kervaire manifold, 77, 86

L

Lefschetz duality, 23
 link of a vertex, 98

M

module
 free, 44
 projective, 44
 stably free, 52

N

Novikov Conjecture, 73
 Novikov Homotopy Invariance Theorem,
 72
 Novikov's Theorem, 54, 55

P

piecewise linear (PL) triangulation, 102
 plumbing, 75, 78

Q

quaternionic line bundle, 58, 63

R

regular cell structure, 16
 regular ring, 96
 Rochlin's Theorem, 61

S

Serre Conjecture, 96
 Serre's Theorem, 57
 simple homotopy equivalence, 105
 splitting theorem, 65
 stable J -homomorphism, 57, 62, 64, 67,
 78, 84
 star of a vertex, 98
 strongly equivalent triangulations, 102
 structure set, 115
 surgery, 81, 84, 85

T

torsion
 of a finitely based acyclic chain complex,
 47, 48
 of an automorphism, 46
 trading handles, 32

W

weakly combinatorial triangulation, 99
 Whitehead group, 39, 40, 95, 96
 Whitehead torsion, 103, 105
 Whitehead's Triangulation Theorem, 14
 Whitney trick, 26
 non-simply connected, 38