

Alexandre Grothendieck: A Mathematical Portrait



Grothendieck

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edited by

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Université Pierre et Marie Curie, Paris



International Press

www.intlpress.com

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Somerville, Massachusetts, U.S.A.

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ISBN: 978-1-57146-282-4

Printed in the United States of America.

18 17 16 15 14 1 2 3 4 5 6 7 8 9

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The frontispiece depicts Grothendieck lecturing in the mid-1960s.
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Foreword

The present book is, at least in part, the outcome of a conference organized in the tumbledown little stone village of Peyresq in the French Alps, entitled *Alexandre Grothendieck: Biography, Mathematics, Philosophy*¹. While there, a group discussion amongst the participants was devoted to the subject of how one could best write a book that would explain, to the generations of young people who didn't even begin to study mathematics until long after Grothendieck had totally disappeared from the scene, what the special and extraordinary nature of his contribution to mathematics really was.

Together, we conceived the idea of a collection of articles, each devoted to one or another aspect of Grothendieck's work. Far from the usual sort of math publication, however, these articles were not to concentrate on the actual mathematical content of Grothendieck's contribution—after all, thousands of pages already exist devoted to all aspects of his work, not least those written by his own hand—but on the features which constitute in some sense his personal mathematical signature. These identifying traits are very familiar to those who know Grothendieck's work: the search for maximum generality, the focus on the harmonious aspects of structure, the lack of interest in special cases, the transfer of attention from objects themselves to morphisms between them, and—perhaps most appealingly—Grothendieck's unique approach to difficulties that consisted in turning them, somehow, upside down, and making them into the actual central point and object of study, an attitude which has the power to subtly change them from annoying obstacles into valuable tools that actually help solve problems and prove theorems.

In order to really comprehend what Grothendieck brought to each subject that he worked in, it helps to have a relatively clear idea of the state of affairs before he came. Placed against their proper background, the fundamental simplicity and the extraordinary power of many of his ideas stand out clearly—unexpected shifts of focus, generalizations to precisely the situations that were classically avoided, new use of an approach or a technique familiar from another domain. Of course, Grothendieck also possessed tremendous technical prowess, not even to mention a capacity for work that led him to concentrate on mathematics for upward of sixteen hours a day in his prime, but those are not the elements that characterize the magic in his style. Rather, it was the absolute simplicity (in his own words, “nobody before me had stooped so low”) and the total freshness and

¹August 24-29, 2008, organized by myself together with Pierre Lochak and Winfried Scharlau

fearlessness of his vision, seemingly unaffected by long-established views and vantage points, that made Grothendieck who he was.

In the years following the Peyresq conference, some of the originally planned contributions to the book fell by the wayside, while some new people—Joe Diestel, David Mumford, Frans Oort, Yuri Manin—came on board to fill lacunae in the mathematical biography. While the end result is far from complete (how could it ever be?), the articles do cover the major aspects of Grothendieck’s work from the topological vector spaces of his graduate student days in the early 1950s through the work on K-theory, schemes, fundamental groups and cohomology of his heyday, to the motives that were still no more than an elusive and enchanting concept governed by a “yoga” based on his powerful intuition in 1970, when, at the age of 42, he brutally left his job at the IHES south of Paris and ruptured with his mathematical companions. Written by people who knew him personally, some by mere acquaintance and others extremely well, some as students and others as colleagues or latecomers to the Grothendieckian scene, the articles in this book contain a wealth of personal memories, explanations and anecdotes about the effect that Grothendieck’s personality, ideas and mathematics had on those who came near him.

Like a cubist work of art, the mathematical portrait of Grothendieck painted in this book is made up of a multiplicity of different planes and different angles. I hope that, taken as a whole, it will make it possible for those who never knew him to discern something of the inimitable character and style of the original.

Leila Schneps
Paris
September 2013