A First Course in Differential Geometry

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To my wife Wenchin Yu

Preface

According to a definition stated by Felix Klein in 1872, we can use geometric transformation groups to classify geometry. The study of properties of geometric figures (curves, surfaces, etc.) that are invariant under a given geometric transformation group G is called the geometry belonging to G. For instance, if G is the projective, affine, or Euclidean group, we have the corresponding projective, affine, or Euclidean geometry.

The differential geometry of a geometric figure F belonging to a group G is the study of the invariant properties of F under G in a neighborhood of an element of F. In particular, the differential geometry of a curve is concerned with the invariant properties of the curve in a neighborhood of one of its points. In analytic geometry the tangent of a curve at a point is customarily defined to be the limit of the secant through this point and a neighboring point on the curve, as the second point approaches the first along the curve. This definition illustrates the nature of differential geometry in that it requires a knowledge of the curve only in a neighborhood of the point and involves a limiting process (a property of this kind is said to be local). These features of differential geometry show why it uses the differential calculus so extensively. On the other hand, local properties of geometric figures may be contrasted with global properties, which require knowledge of entire figures.

The origins of differential geometry go back to the early days of the differential calculus, when one of the fundamental problems was the determination of the tangent to a curve. With the development of the calculus, additional geometric applications were obtained. The principal contributors in this early period were Leonhard Euler (1707–1783), Gaspard Monge (1746–1818), Joseph Louis Lagrange (1736–1813), and Augustin Cauchy (1789–1857). A decisive step forward was taken by Karl Friedrich Gauss (1777–1855) with his development of the intrinsic geometry on a surface. This idea of Gauss was generalized to $n(>3)$-dimensional space by Bernhard Riemann (1826–1866), thus giving rise to the geometry that bears his name.

This book is designed to introduce differential geometry to beginning graduate students as well as advanced undergraduate students (this introduction in the latter case is important for remedying the weakness of geometry in the usual undergraduate curriculum). In the last couple of decades differential geometry, along with other branches of mathematics, has been highly developed. In this book we will study only the traditional topics, namely, curves and surfaces in a three-dimensional Euclidean space E^3. Unlike most classical books on the subject, however, more attention is paid here to the relationships between local and global properties, as opposed to local properties only. Although we restrict our attention to curves and surfaces in E^3, most global theorems for curves and surfaces in this book can be extended to either higher dimensional spaces or more general curves and surfaces or both. Moreover, geometric interpretations are given along with analytic expressions. This will enable students to make use of geometric intuition, which is a precious tool for studying geometry and related problems; such a tool is seldom encountered in other branches of mathematics.

We use vector analysis and exterior differential calculus. Except for some tensor conventions to produce simplifications, we do not employ tensor calculus, since there is no benefit in its use for our study in space E^3. There are four chapters whose contents are, briefly, as follows.

Chapter 1 contains, for the purpose of review and for later use, a collection of fundamental material taken from point-set topology, advanced calculus, and linear algebra. In keeping with this aim, all proofs of theorems are self-contained and all theorems are expressed in a form suitable for direct later application. Probably most students are familiar with this material except for Section 6 on differential forms.

In Chapter 2 we first establish a general local theory of curves in E^3, then give global theorems separately for plane and space curves, since those for plane curves are not special cases of those for space curves. We also prove one of the fundamental theorems in the local theory, the uniqueness theorem for curves in E^3. A proof of this existence theorem is given in Appendix 1.

Chapter 3 is devoted to a local theory of surfaces in E^3. For this theory we only state the fundamental theorem (Theorem 7.3), leaving the proofs of the uniqueness and existence parts of the theorem to, respectively, Chapter 4 (Section 4) and Appendix 2.

Chapter 4 begins with a discussion of orientation of surfaces and surfaces of constant Gaussian curvature, and presents various global theorems for surfaces.

Most sections end with a carefully selected set of exercises, some of which supplement the text of the section; answers are given at the end of the book. To allow the student to work independently of the hints that accompany some of the exercises, each of these is starred and the hint together with the answer appear at the end of the book. Numbers in brackets refer to the items listed in the Bibliography at the end of the book.

Two enumeration systems are used to subdivide sections; in Chapters 1 (except Sections 4 and 7) and 2, triple numbers refer to an item (e.g., a theorem or definition), whereas in Chapters 3 and 4 such an item is referred to by a double number. However, there should be no difficulty in using the book for reference purposes, since the title of the item is always written out (e.g., Corollary 5.1.6 of Chapter 1 or Lemma 1.5 of Chapter 3).

This book can be used for a full-year course if most sections of Chapter 1 are studied thoroughly.

For a one-semester course I suggest the use of the following sections:

Chapter 1: Sections 3.1, 3.2, 3.3, 6.

Chapter 2: Section 1.1 (omit 1.1.4–1.1.6), Section 1.2 (omit 1.2.6, 1.2.7), Section 1.3 (omit 1.3.7–1.3.12), Sections 1.4 and 1.5 (omit 1.5.5); Section 2 (omit 2.3, 2.5, 2.6.4–2.6.6, 2.9–2.11, 2.14–2.23); Section 3 (omit 3.1.8–3.1.14).

Chapter 3: Section 1 (omit the proof of 1.6, 1.7, 1.8, the proof of 1.10, 1.11–1.13, 1.15–1.18); Section 2 (omit the proof of 2.4); Sections 3–9; Section 10 (omit the material after 10.7).

Chapter 4: Section 1 (omit the proofs of 1.3 and 1.4); Section 3 (omit 3.14); Sections 4 and 5.

For a course lasting one quarter I suggest omission of the following material from the one-semester outline above: Chapter 2: the second proof of 2.6, 3.2; Chapter 3: the details of 1.3 and 1.4, the proof of 5.7, Section 6, the proofs of 8.1 and 8.2; Chapter 4: Section 5.

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Bethlehem, Pennsylvania
September, 1980

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