Surveys of Modern Mathematics
Volume VI

Open Problems and Surveys of Contemporary Mathematics

edited by

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Open Problems and Surveys of Contemporary Mathematics

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SURVEYS OF MODERN MATHEMATICS

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Preface

This book consists mainly of lecture notes of some talks and courses at Mathematical Sciences Center (MSC) of Tsinghua University together with several other papers.

Since it was founded in December 2009, one of the missions of MSC has been to teach both undergraduate and graduate students important ideas, theories and results of contemporary mathematics. These lecture notes reflect this philosophy. They are expository and accessible to both students and nonexperts. On the other hand, they also contain novel ideas or presentations of important topics in mathematics. Therefore, this book is also useful to experts. Especially we would like to point out that the last paper in this book *Open Problems in Differential Geometry* by the third editor of this book is only the first three of many lectures given by him in both Beijing and Taipei, which can be considered as a reviewing and updating of the very influential open problem lists by him.

Besides these lecture notes from MSC, this book also contains four other papers. The first is a paper by James Milne based on his talk at the seminar "What is ..." at University of Michigan. The concept of motives is important and difficult, and the talk and this paper are attempts by an expert to explain it in concrete terms. The second is a master thesis in 2002 by Joris van Hoboken who gives a coherent and accessible exposition of the ubiquity of the important ADE classification in mathematics, which originally occurred in the classification of simple complex Lie algebras. Joris van Hoboken switched to study law right after obtaining his Master degree and is now a senior researcher at a law school. The ADE classification occurs at many different situations, and it is still a mystery whether there are some deep, intrinsic connections between them. This master thesis was never published and has been highly cited and circulated on the web. We are grateful that Dr. van Hoboken has given us permission to include it in the current book. We hope that this will make the ADE classification better known to the reader and also give a permanent record of this beautiful master thesis. The other two are reprints of papers of the third editor. The short paper *A note on the distribution of critical points of eigenfunctions* considered a novel question. As it is well-known, the location and distribution of the zero sets (i.e., nodal sets) of eigenfunctions of Riemannian manifolds have been extensively and intensively studied. Critical points of eigenfunctions are also special and deserve to be understood better. Analysis on nonsmooth spaces has been becoming quite important and applied to several subjects in mathematics. The paper is one of the early papers in this subject.[1] Due to inaccessibility and no review of it in MathSciNet, this paper has been largely unknown. We hope that its inclusion in this book will be valuable to the reader as well.

It has been a lot of work for the speakers at MSC to write up their lecture notes. We would like to thank them, especially the four note-takers and co-authors (Hui Ma, Chun-Jun Tsai, Mu-Tao Wang, En-Tao Zhao) of the last paper in this

[1]For a recent survey, see J. Heinonen, *Nonsmooth calculus.* Bull. Amer. Math. Soc. (N.S.) 44 (2007), no. 2, 163–232.

book, for their efforts and contributions. We would also like to thank reviewers of the papers in this book for their help.

This book marks the beginning of publication from MSC and we hope and expect that future volumes will appear regularly.

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