Surveys in Differential Geometry

Vol. 1: Lectures given in 1990
edited by S.-T. Yau and H. Blaine Lawson

Vol. 2: Lectures given in 1993
edited by C.C. Hsiung and S.-T. Yau

Vol. 3: Lectures given in 1996
edited by C.C. Hsiung and S.-T. Yau

Vol. 4: Integrable systems
edited by Chuu Lian Terng and Karen Uhlenbeck

Vol. 5: Differential geometry inspired by string theory
edited by S.-T. Yau

Vol. 6: Essays on Einstein manifolds
edited by Claude LeBrun and McKenzie Wang

Vol. 7: Papers dedicated to Atiyah, Bott, Hirzebruch, and Singer
edited by S.-T. Yau

Vol. 8: Papers in honor of Calabi, Lawson, Siu, and Uhlenbeck
edited by S.-T. Yau

Vol. 9: Eigenvalues of Laplacians and other geometric operators
edited by A. Grigor'yan and S-T. Yau

Vol. 10: Essays in geometry in memory of S.-S. Chern
edited by S.-T. Yau

Vol. 11: Metric and comparison geometry
edited by Jeffrey Cheeger and Karsten Grove

Vol. 12: Geometric flows
edited by Huai-Dong Cao and S.-T. Yau

Vol. 13: Geometry, analysis, and algebraic geometry
edited by Huai-Dong Cao and S.-T.Yau

Vol. 14: Geometry of Riemann surfaces and their moduli spaces
edited by Lizhen Ji, Scott A. Wolpert, and S.-T. Yau

Vol. 15: Perspectives in mathematics and physics: Essays dedicated to Isadore Singer
edited by Tomasz Mrowka and S.-T. Yau

Vol. 16: Geometry of special holonomy and related topics
edited by Naichung Conan Leung and S.-T. Yau

Vol. 17: In Memory of C. C. Hsiung: Lectures given in 2010
edited by Huai-Dong Cao and S.-T. Yau

Vol. 18: Geometry and Topology: Lectures given 2011 and 2012
edited by Huai-Dong Cao and S.-T. Yau

Volume XVIII

Surveys in Differential Geometry

Geometry and Topology

Lectures given at the Geometry and Topology conferences at Harvard University in 2011 and at Lehigh University in 2012

edited by

Huai-Dong Cao
and Shing-Tung Yau

International Press
www.intlpress.com

Series Editor: Shing-Tung Yau

Surveys in Differential Geometry, Vol. 18 (2013)
Geometry and Topology: Lectures given at the Geometry and Topology conferences at Harvard University in 2011 and at Lehigh University in 2012

Volume Editors:
Huai-Dong Cao (Lehigh University)
Shing-Tung Yau (Harvard University)

Copyright © 2013 by International Press
Somerville, Massachusetts, U.S.A.

All rights reserved. Individual readers of this publication, and non-profit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgement of the source is given. Republication, systematic copying, or mass reproduction of any material in this publication is permitted only under license from International Press.

Excluded from these provisions is material in articles to which the author holds the copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author. (Copyright ownership is indicated in the notice on the first page of each article.)

ISBN 978-1-57146-269-5

Printed in the United States of America.

17 16 15 14 13 1 2 3 4 5 6 7 8 9

Preface

Each year the *Journal of Differential Geometry* (JDG) sponsors a conference on Geometry and Topology. The conference is held every third year at Harvard University, and other years at Lehigh University.

The current volume includes papers presented by several speakers at both the 2011 conference at Harvard and the 2012 conference at Lehigh. We have articles by Simon Brendle, on the Lagrangian minimal surface equation and related problems; by Sergio Cecotti and Cumrun Vafa, concerning classification of complete $N = 2$ supersymmetric theories in 4 dimensions; by F. Reese Harvey and H. Blaine Lawson Jr., on existence, uniqueness, and removable singularities for non-linear PDEs in geometry; by János Kollár, concerning links of complex analytic singularities; by Claude LeBrun, on Calabi energies of extremal toric surfaces; by Mu-Tao Wang, concerning mean curvature flows and isotopy problems; and by Steve Zelditch, on eigenfunctions and nodal sets.

We are grateful to the many distinguished geometers and topologists who presented invited talks at these two conferences, especially those who contributed articles to this volume of the *Surveys in Differential Geometry* book series.

Huai-Dong Cao
Lehigh University

Shing-Tung Yau
Harvard University

Contents

Preface v

On the Lagrangian minimal surface equation and related problems
Simon Brendle 1

Classification of complete $\mathcal{N} = 2$ supersymmetric theories in 4 dimensions
Sergio Cecotti and Cumrun Vafa 19

Existence, uniqueness and removable singularities for nonlinear partial differential equations in geometry
F. Reese Harvey and H. Blaine Lawson, Jr. 103

Links of complex analytic singularities
János Kollár 157

Calabi energies of extremal toric surfaces
Claude LeBrun 195

Mean curvature flows and isotopy problems
Mu-Tao Wang 227

Eigenfunctions and nodal sets
Steve Zelditch 237