

Index Theory

with Applications to
Mathematics and
Physics

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David D. Bleecker
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To David

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Synopsis

Preface. Target Audience and Prerequisites. Outline of History. Further Reading. Questions of Style. Acknowledgments and Dedication.

Chapter 1. Fredholm Operators. Hierarchy of Mathematical Objects. The Concept of a Bounded Fredholm Operator in Hilbert Space. Algebraic Properties. Operators of Finite Rank. The Snake Lemma of Homological Algebra. Product Formula. Operators of Finite Rank and the Fredholm Integral Equation. The Spectra of Bounded Linear Operators (Terminology).

Chapter 2. Analytic Methods. Compact Operators. Adjoint and Self-Adjoint Operators - Recalling Fischer-Riesz. Dual characterization of Fredholm operators: either by finite-dimensional kernel and cokernel, or by finite-dimensional kernels of the operator and its adjoint operator *and* closed image. Compact Operators: Spectral Decomposition, Why Compact Operators also are Called *Completely Continuous*, \mathcal{K} as Two-Sided Ideal, Closure of Finite-Rank Operators, and Invariant under $*$. Classical Integral Operators. Fredholm Alternative and Riesz Lemma. Sturm-Liouville Boundary Value Problems. Unbounded Operators: Comprehensive Study of Linear First Order Differential Operators Over S^1 : Sobolev Space, Dirac Distribution, Normalized Integration Operator as *Parametrix*, The Index Theorem on the Circle for Systems. Closed Operators, Closed Extensions, Closed (not necessarily bounded) Fredholm Operators, Composition Rule, Symmetric and Self-Adjoint Operators, Formally Self-Adjoint and Essentially Self-Adjoint. Spectral Theory. Metrics on the Space of Closed Operators. Trace Class and Hilbert-Schmidt Operators.

Chapter 3. Fredholm Operator Topology. Calkin Algebra and Atkinson's Theorem. Perturbation Theory: Homotopy Invariance of the Index, Homotopies of Operator-Valued Functions, The Theorem of Kuiper. The Topology of \mathcal{F} : The Homotopy Type, Index Bundles, The Theorem of Atiyah-Jänich. Determinant Line Bundles: The Quillen Determinant Line Bundle, Determinants, The Segal-Furutani Construction. Spectral Invariants: Essentially Unitary Equivalence, What Is a Spectral Invariant? Eta Function, Zeta Function, Zeta Regularized Determinant.

Chapter 4. Wiener-Hopf Operators. The Reservoir of Examples of Fredholm Operators. Origin and Fundamental Significance of Wiener-Hopf Operators. The *Characteristic Curve* of a Wiener-Hopf Operator. Wiener-Hopf Operators and Harmonic Analysis. The Discrete Index Formula. Noether's Theorem for the Hilbert Transform. The Case of Systems. The Continuous Analogue.

Chapter 5. Partial Differential Equations in Euclidean Space, Revisited. Review of Classical Linear Partial Differential Equations: Constant and Variable Coefficients, Wave Equation, Heat Equation, Laplace Equation, Characteristic Polynomial. Elliptic Differential Equations: Where Do Elliptic Differential Operators Arise? Boundary-Value Conditions. Main Problems of Analysis and the Index Problem. Calculations. Elementary Examples. The Noether(-Hellwig-Vekua) Problem with Nonvanishing Index.

Chapter 6. Differential Operators over Manifolds. Motivation. Differentiable Manifolds - Foundations: Tangent Space. Cotangent Space. Geometry of C^∞ Mappings: Embeddings, Immersions, Submersions, Embedding Theorems. Integration on Manifolds: Hypersurfaces, Riemannian Manifolds, Geodesics, Orientation. Exterior Differential Forms and Exterior Differentiation. Covariant Differentiation, Connections and Parallelity: Connections on Vector Bundles, Parallel Transport, Connections on the Tangent Bundle, Clifford Modules and Operators of Dirac Type. Differential Operators on Manifolds and Symbols: Our Data, Symbolic Calculus, Formal Adjoints. Elliptic Differential Operators. Definition and Standard Examples. Manifolds with Boundary.

Chapter 7. Sobolev Spaces (Crash Course). Motivation. Equivalence of Different Local Definitions. Various Isometries. Global, Coordinate-Free Definition. Embedding Theorems: Dense Subspaces; Truncation and Mollification; Differential Embedding; Rellich Compact Embedding. Sobolev Spaces Over Half Spaces. Trace Theorem. Case Studies: Euclidean Space and Torus; Counterexamples.

Chapter 8. Pseudo-Differential Operators. Motivation: Fourier Inversion; Symbolic Calculus; Quantization. *Canonical* and *Principally Classical* Pseudo-Differential Operators. Pseudo-Localilty; Singular Support. Standard Examples: Differential Operators; Singular Integral Operators. Oscillatory Integrals. Kuranishi Theorem. Change of Coordinates. Pseudo-Differential Operators on Manifolds. Graded $*$ -Algebra. Invariant Principal Symbol; Exact Sequence; Noncanonical Op-Construction as Right Inverse. Coordinate-Free (Truly Global) Approach: Bokobza-Haggiag-Fourier Transformation; Bokobza-Haggiag Amplitudes; Bokobza-Haggiag Invertible Op-Construction; Approximation of Differential Operators.

Chapter 9. Elliptic Operators over Closed Manifolds. Continuity of Pseudo-Differential Operators between Sobolev Spaces. Parametrices for Elliptic Operators: Regularity and Fredholm Property. Topological Closures. Outer Tensor Product on Product Manifolds. The Topological Meaning of the Principal Symbol (Simple Case Involving Local Boundary Conditions).

Chapter 10. Introduction to Topological K-Theory. Winding Numbers. One-Dimensional Index Theorem. Counter-Intuitive Dimension Two: Bending a Plane. The Topology of the General Linear Group. The Grothendieck Ring of Vector Bundles. K-Theory with Compact Support. Proof of the Bott Periodicity Theorem.

Chapter 11. The Index Formula in the Euclidean Case. Index Formula and Bott Periodicity: Three Integer Invariants. The Difference Bundle of an Elliptic Operator: Operators Equal to the *Identity at Infinity*; Complexes of Vector Bundles

with Compact Support; Symbol Class in K -Theory with Compact Support. The Index Theorem for $\text{Ell}_c(\mathbb{R}^n)$.

Chapter 12. The Index Theorem for Closed Manifolds. K -Theoretic Proof of the Index Theorem by Embedding: Pilot Study — The Index Theorem for Embeddings with Trivial Normal Bundle. Proof of the Index Theorem for Non-Trivial Normal Bundle: The Difference Element Construction, Revisited; Symbol Class; Thom Isomorphism of K -Theory; Definition of the Topological Index; Definition of the Analytic Index; Foundations of Equivariant K -Theory. Multiplicative Property: Formulation; How it Fits into the Embedding Proof; Proof of the Multiplicative Property. Short Comparison of the Cobordism, the Embedding and the Heat Equation Proof; Outlook to Spectral Theory, Asymmetry, and Inverse Problems.

Chapter 13. Classical Applications (Survey). General Appraisal. Cohomological Formulation of the Index Formula: Comparison of K -Theoretic and Cohomological Thom Isomorphisms; Orientation Class; Chern Classes; Chern Character; Todd Class; Chern Character Defect. The Case of Systems (Trivial Bundles). Examples of Vanishing Index. Euler Number and Signature. Vector Fields on Manifolds. Abelian Integrals and Riemann Surfaces. The Theorem of Riemann-Roch-Hirzebruch. The Index of Elliptic Boundary-Value Problems. Real Operators. The Lefschetz Fixed-Point Formula. Analysis on Symmetric Spaces. Further Applications.

Chapter 14. Physical Motivation and Overview. Mode of Reasoning in Physics. String Theory and Quantum Gravity. The Experimental Side. Classical Field Theory: Newton-Maxwell-Lorentz, Faraday 2-Form, Abstract Flat Minkowski Space-Time, Relativistic Mass, Relativistic Kinetic Energy, Inertial System, Lorentz Transformations and Poincaré Group, Relativistic Deviation from Flatness, *Twin Paradox*, Variational Principles. Kaluza-Klein Theory: Simultaneous Geometrization of Electro-Magnetism and Gravity, Other *Grand Unified Theories*, String Theory. Quantum Theory: Photo-Electric Effect, Atomic Spectra, Quantizing Energy, State Spaces of Systems of Particles, Basic Interpretive Assumptions. Heisenberg Uncertainty Principle. Evolution with Time - The Schrödinger Picture. Nonrelativistic Schrödinger Equation and Atomic Phenomena. *Minimal Replacement* and Covariant Differentiation. Anti-Particles and Negative-Energy States. Unreasonable Success of the Standard Model. Dirac Operator vs. Klein-Gordon Equation. Feynman Diagrams.

Chapter 15. Geometric Preliminaries. Principal G -Bundles; Hopf Bundle. Connections and Curvature: Connection 1-Form; Maurer-Cartan Form; Horizontal Lift. Equivariant Forms and Associated Bundles: Associated Vector Bundles, Equivariance, and Basic Forms; Horizontal Equivariant Forms; Covariant Differentiation and the General Bianchi Identity; Inner Products, Hodge Star Operator, and Formal Adjoints; Inner Products, Hodge Star Operator, and Formal Adjoints. Gauge Transformations: Distinguishing Gauge Transformations from Automorphisms; The Group of Gauge Transformations; The Action of Gauge Transformations on Connections; Lie Algebra Analogy and Infinitesimal Action. Curvature in Riemannian Geometry: The Bundle of Linear Frames; Connections and Forms;

Kozul Connection; The Orthonormal Frame Bundle; Metric Connections; The Fundamental Lemma of Riemannian Geometry and the Levi-Civita Connection; Local Coordinates and Christoffel Symbols; The Curvature of the Levi-Civita Connection; First and Second Bianchi Identities; Ricci and Scalar Curvature as Contractions, and Einstein's Equation; All Possible Curvature Tensors on \mathbb{R}^n and the Kulkarni-Nomizu Product; Curvature Parts on 4-Manifolds and Self-Duality. Bochner-Weitzenböck Formulas: Fibered Products; Contractions and Components; The Connection Laplacian, the Hodge Laplacian, and the Bochner-Weitzenböck Formula; Special Cases. Characteristic Classes and Curvature Forms: Chern Classes as Curvature Forms; The Pfaffian; Pontryagin Classes; Other Characteristic Classes Related to Index Theory; Multiplicative Classes; Todd Class and L -Polynomials; Recalculating Characteristic Classes; Unifications on Almost-Complex Manifolds. Holonomy.

Chapter 16. Gauge Theoretic Instantons. The Yang-Mills Functional. Instantons on Euclidean 4-Space. Linearization of the Manifold of Moduli of Self-dual Connections. Manifold Structure for Moduli of Self-dual connections.

Chapter 17. The Local Index Theorem for Twisted Dirac Operators. Clifford Algebras and Spinors: Clifford Algebra Basics; Spin Groups and Double Cover; Spinor Representations; Supertrace. Spin Structures and Twisted Dirac Operators: Čech Cohomology; Admittance of Spin Structures; Standard and Twisted Dirac Operators; Chirality. The Spinorial Heat Kernel: Index, Spectral Asymmetry and the Existence of the Heat Kernel; Solving the Spinorial Heat Equations; Calculating Index and Supertrace; General Heat Kernels. The Asymptotic Formula for the Heat Kernel: Why Asymptotic Expansion? The Radial Gauge; About the Geometry of the Ball; Further Approximations. The Local Index Formula: Content and Meaning of the Local Index Formula; How the Curvature Terms Arise in the Heat Asymptotics; The case $m = 1$ (surfaces); The case $m = 2$ (4-manifolds); Proof of the Local Index Formula for Arbitrary Even Dimensions; Index Theorem for Twisted Dirac Operators; \hat{A} Genus; Rokhlin's Theorem. The Index Theorem for Standard Geometric Operators: Index Theorem for Generalized Dirac Operators; Twisted Generalized Dirac Operators; The Hirzebruch Signature Formula; The Gauss-Bonnet-Chern Formula; The Generalized Yang-Mills Index Theorem; The Hirzebruch-Riemann-Roch Formula for Kähler Manifolds.

Chapter 18. Seiberg-Witten Theory. Background and Survey: Intersection Form and Homotopy Type of Compact Oriented Simply-Connected Four-Manifolds; Unimodular Forms and Freedman's Theorem; Existence of Differentiable Structures; Donaldson's Polynomial Invariants; Results Concerning Symplectic Manifolds; Purely Geometric Applications. Spin^c Structures and the Seiberg-Witten Equations: Admittance of Spin^c Structures, Spin^c Dirac Operators; Motivating and Defining the Unperturbed and the Perturbed Seiberg-Witten Equations. Generic Regularity of the Moduli Spaces: Gauge Transformations; Moduli Space of Solutions of the Perturbed S-W Equations; The Formal Dimension of the Moduli Space; Seiberg-Witten Function; Quotient Manifolds; Manifold Structure for the Parametrized Moduli Space; Generic Regularity; A Priori Bounds; Sobolev Estimates; Compactness of Moduli Spaces; Definition of the S-W Invariant; Metric Dependence of Connections and Dirac Operators; Oriented Cobordism; Fredholm Transversality; Full Invariance of the S-W Invariant.

Appendix A. Fourier Series and Integrals (fundamental principles).

Fourier Series: The Fundamental Function Spaces on S^1 ; Density; Orthonormal Basis; Fourier Coefficients; Plancherel's Identity; Product and Convolution. The Fourier Integral: Different Integral Conventions; Duality Between Local and Global - Point and Neighborhood - Multiplication and Differentiation - Bounded and Continuous; Fourier Inversion Formula; Plancherel and Poisson Summation Formulae; Parseval's Equality; Higher Dimensional Fourier Integrals.

Appendix B. Vector Bundles. Basic Definitions and First Examples. Ho-

motopy Equivalence and Isomorphy. Clutching Construction and Suspension.

Bibliography. Key References. Classical and Recent Textbooks. References

to Technical Details; History; Perspectives.

TABLE 0.1. Suggested packages (selections, *curricula*) for upper-undergraduate and graduate classes/seminars and teach-yourself

Aims	Logical Order by Boxed Chapter Numbers
Index Theorem and Topolog. K -Theory	$\boxed{\text{App. B}} \rightarrow \boxed{1.1-1.3, 2.1,2.2, 3.1-3.8, \text{Thm. 5.11}} \rightarrow \boxed{6-7}$ $\rightarrow \boxed{8.5,9} \rightarrow \boxed{10-12.2} \rightarrow \boxed{13.1-13.5, 13.10,13.11} \rightarrow \boxed{18.1}$
Index Theorem via Heat Equation	$\boxed{\text{App. A}} \rightarrow \boxed{1.2,3.3} \rightarrow \boxed{5.2, 6-7, 8.5} \rightarrow \boxed{8.5,9} \rightarrow \boxed{12.3}$ $\rightarrow \boxed{15} \rightarrow \boxed{17}$
Gauge-Theoretic Physics	$\boxed{14} \rightarrow \boxed{5-6} \rightarrow \boxed{9.2} \rightarrow \boxed{12.3} \rightarrow \boxed{13.8,13.11} \rightarrow \boxed{15} \rightarrow \boxed{16}$ $\rightarrow \boxed{19}$
Spectral Geometry	$\boxed{1-4} \rightarrow \boxed{6,15} \rightarrow \boxed{12.3} \rightarrow \boxed{13.4-13.11} \rightarrow \boxed{17.4-17.6} \rightarrow \boxed{18}$
Global and Micro-Local Analysis	$\boxed{\text{App. A,B}} \rightarrow \boxed{1, 2, 3.1-3.5} \rightarrow \boxed{3.10,4} \rightarrow \boxed{5-6} \rightarrow \boxed{14-15}$ $\rightarrow \boxed{7-9} \rightarrow \boxed{10.1, 10.2} \rightarrow \boxed{12.3,13} \rightarrow \boxed{16,17.6,18}$

Preface

Target Audience and Prerequisites. The mathematical philosophy of index theory and all its basic concepts, technicalities and applications are explained in Parts I-III. Those are the easy parts. They are written for upper undergraduate students or graduate students to bridge the gap between rule-based learning and first steps towards independent research. They are also recommended as general orientation to mathematics teachers and other senior mathematicians with different background. All interested can pick up a single chapter as bedside reading.

In order to enjoy reading or even work through Parts I-III, we expect the reader to be familiar with the concept of a smooth function and a complex separable Hilbert space. Nothing more — but a will to acquire specialized topics in functional analysis, algebraic topology, elliptic operator theory, global analysis, Riemannian geometry, complex variables, and some other subjects. Catching so many different concepts and fields can make the first three Parts a bit sophisticated for a busy reader. Instead of ascending systematically from simple concepts to complex ones in the classical Bourbaki style, we present a patch-work of definitions and results when needed. In each chapter we present a couple of fully comprehensible, important, deep mathematical *stories*. That, we hope, is sufficient to catch our four messages:

- (1) Index theory is about *regularization*, more precisely, the index quantifies the defect of an equation, an operator, or a geometric configuration from being regular.
- (2) Index theory is also about *perturbation invariance*, i.e., the index is a meaningful quantity stable under certain deformations and apt to store certain topological or geometric information.
- (3) Most important for many mathematicians, the index *interlinks quite diverse mathematical fields*, each with its own very distinct research tradition.
- (4) Index theory *trains the student* to recognize all the elementary topics of linear algebra in finite dimensions in the sophisticated topics of infinite-dimensional and nonlinear analysis and geometry.

Part IV is different. It is also self-contained. Choosing one or two chapters of this Part IV of the book would make a suitable text for a graduate course in selected topics of global analysis. All concepts will be explained fully and rigorously, but much shorter than in the first Parts. This last Part is written for graduate students, PhD students and other experienced learners, interested in low-dimensional topology and gauge-theoretic particle physics. We try to explain the very place of index theory in *geometry* and for revisiting *quantum field theory*. There are thousands of other calculations, observations and experiments. But there is something special about the actual and potential contributions of index theory. Index theory

is about chirality (asymmetry) of zero modes in the spectrum and classifies connections (back ground fields) and a variety of other intrinsic properties in geometry and physics. It is not just about some more calculations, some more numbers and relations.

Outline of History. When first considering infinite-dimensional linear spaces, there is the immediate realization that there are injective and surjective linear endomorphisms which are not isomorphisms, and more generally the dimension of the kernel minus that of the cokernel (i.e., the index) could be any integer. However, in the classical theory of Fredholm integral operators which goes back at least to the early 1900s (see [144]), one is dealing with compact perturbations of the identity and the index is zero. FRITZ NOETHER (in his study [321] of singular integral operators and the oblique boundary problem for harmonic functions, published in 1920), was the first to encounter the phenomenon of a nonzero index for operators naturally arising in analysis *and* to give a formula for the index in terms of a winding number constructed from data defining the operator. Over some decades, this result was expanded in various directions by G. HELLWIG, I.N. VEKUA and others (see [422]), contrary to R. COURANT's and D. HILBERT's expectation in [113] that "linear problems of mathematical physics which are correctly posed behave like a system of N linear algebraic equations in N unknowns", i.e., they should satisfy the *Fredholm alternative* and always yield vanishing index. Meanwhile, many working mainly in abstract functional analysis were producing results, such as the stability of the index of a Fredholm operator under perturbations by compact operators or bounded operators of sufficiently small operator norm (e.g., first J.A. DIEUDONNÉ [117], followed by F.V. ATKINSON [49], B. YOOD [446], I.Z. GOHBERG and M.G. KREIN [175], etc.).

Around 1960, the time was ripe for I.M. GELFAND [157] to propose that the index of an elliptic differential operator (with suitable boundary conditions in the presence of a boundary) should be expressible in terms of the coefficients of highest order part (i.e., the principal symbol) of the operator, since the lower order parts provide only compact perturbations which do not change the index. Indeed, a continuous, ellipticity-preserving deformation of the symbol should not affect the index, and so GELFAND noted that the index should only depend on a suitably defined homotopy class of the principal symbol. The hope was that the index of an elliptic operator could be computed by means of a formula involving only the topology of the underlying domain (the manifold), the bundles involved, and the symbol of the operator. In early 1962, M.F. ATIYAH and I.M. SINGER discovered the (elliptic) Dirac operator in the context of Riemannian geometry and were busy working at Oxford on a proof that the \hat{A} -genus of a spin manifold is the index of this Dirac operator. At that time, S. SMALE happened to pass through Oxford and turned their attention to GELFAND's general program described in [157]. Drawing on the foundational and case work of analysts (e.g., M.S. AGRANOVICH, A.S. DYNIN, L. NIRENBERG, R.T. SEELEY and A.I. VOLPERT), particularly that involving pseudo-differential operators, ATIYAH and SINGER could generalize HIRZEBRUCH's proof of the Hirzebruch-Riemann-Roch theorem of 1954 (see [204]) and discovered and proved the desired index formula at Harvard in the Fall of 1962. Moreover, the Riemannian Dirac operator played a major role in establishing the general case. The details of this original proof involving cobordism actually first appeared in [325]. A K -theoretic embedding proof was given in [44], the first in a series of five papers.

This proof was more direct and susceptible to generalizations (to G -equivariant elliptic operators in [42] and families of elliptic operators in [47]).

The proof of the Index Theorem in [44] was inspired by GROTHENDIECK's proof and thorough generalization of the Hirzebruch-Riemann-Roch Theorem, explained in [83]. We shall present the approach in detail in Chapters 10-12 of this book. The invariance of the index under homotopy implies that the index (say, the *analytic index*) of an elliptic operator is stable under rather dramatic, but continuous, changes of its principal symbol while maintaining ellipticity. Using this fact, one finds (after considerable effort) that the analytical index of an elliptic operator transforms predictably under various global operations such as embedding and extension. Using K -theory and Bott periodicity, a topological invariant (say, the *topological index*) with the same transformation properties under these global operations is constructed from the symbol of the elliptic operator. One then verifies that a general index function having these properties is unique, subject to normalization. To deduce the Atiyah-Singer Index Theorem (i.e., *analytic index* = *topological index*), it then suffices to check that the two indices are the same in the trivial case where the base manifold is just a single point. A particularly nice exposition of this approach for twisted Dirac operators over even-dimensional manifolds (avoiding many complications of the general case) is found in E. GUENTNER's article [191] following an argument of P. BAUM.

Not long after the K -theoretical embedding proof (and its variants), there emerged a fundamentally different means of proving the Atiyah-Singer Index Theorem, namely the *heat kernel method*. This is worked out here (see Chapter 17 in the important case of the chiral half \mathcal{D}^+ of a twisted Dirac operator \mathcal{D} . In the index theory of closed manifolds, one usually studies the index of a chiral half \mathcal{D}^+ instead of the total Dirac operator \mathcal{D} , since \mathcal{D} is symmetric for compatible connections and then $\text{index } \mathcal{D} = 0$.) The heat kernel method had its origins in the late 1960s (e.g., in [288], inspired by [299] of 1949) and was pioneered in the works [328], [163], [33]. In the final analysis, it is debatable as to whether this method is really much shorter or better. This depends on the background and taste of the beholder. Geometers and analysts (as opposed to topologists) are likely to find the heat kernel method appealing. The method not only applies to geometric operators which are expressible in terms of twisted Dirac operators, but also largely for more general elliptic pseudo-differential operators, as R.B. MELROSE has done in [289]. Moreover, the heat method gives the index of a "geometric" elliptic differential operator naturally as the integral of a characteristic form (a polynomial of curvature forms) which is expressed solely in terms of the geometry of the operator itself (e.g., curvatures of metric tensors and connections). One does not destroy the geometry of the operator by using ellipticity-preserving deformations. Rather, in the heat kernel approach, the invariance of the index under changes in the geometry of the operator is a consequence of the index formula itself more than a means of proof. However, considerable analysis and effort are needed to obtain the heat kernel for $e^{-t\mathcal{D}^2}$ and to establish its asymptotic expansion as $t \rightarrow 0^+$. Also, it can be argued that in some respects the K -theoretical embedding/cobordism methods are more forceful and direct. Moreover, in [270], we are cautioned that the index theorem for families (in its strong form) generally involves torsion elements in K -theory that are not detectable by cohomological means, and hence are not computable in terms of local densities produced by heat asymptotics. Nevertheless, when this

difficulty does not arise, the K-theoretical expression for the topological index may be less appealing than the integral of a characteristic form, particularly for those who already understand and appreciate the geometrical formulation of characteristic classes. More importantly, the heat kernel approach exhibits the index as just one of a whole sequence of spectral invariants appearing as coefficients of terms of the asymptotic expansion (as $t \rightarrow 0^+$) of the trace of the relevant heat kernel. (On p. 118, we guide the reader to the literature about these particular spectral invariants and their meaning in modern physics. The required mathematics for that will be developed in Section 17.4.) All disputes aside, the student who learns *both* approaches and formulations to the index formula will be more accomplished (and probably a good deal older).

Further Reading. What the coverage of topics in this book is concerned, we hope our table of contents needs no elaboration, except to say that space limitations prevented the inclusion of some important topics (e.g., the index theorem for families; index theory for manifolds with boundary, other than the Atiyah-Patodi-Singer Theorem; L^2 -index theory and coarse geometry of noncompact manifolds; R. NEST's and B. TSYGAN's algebraic and operator theoretic index theory of [314, 315]; P. KRONHEIMER's and T. MROWKA's visionary work on knot homology groups from instantons; lists of all calculated spectral invariants; aspects of analytic number theory). However, we now provide some guidance for further study. A fairly complete exposition, by ATIYAH himself, of the history of index theory from 1963 to 1984 is found in Volume 3 of [25] and duplicated in Volume 4. Volumes 3, 4 and 5 contain many unsurpassed articles written by ATIYAH and collaborators on index theory and its applications to gauge theory. In the informative — and charming [445], S.-T. YAU collected *The founders of index theory: reminiscences of and about Sir Michael Atiyah, Raoul Bott, Friedrich Hirzebruch, and I. M. Singer*. N. HITCHIN's short text [213] on the 2004 Abel Prize Laureates describes the index theorem, where it came from, its different manifestations and a collection of applications. It indicates how one can use the theorem as a tool in a concrete fashion without necessarily retreating into the details of the proof. We all owe a debt of gratitude to H. SCHRÖDER for the definitive guide to the literature on index theory (and its roots and offshoots) through 1994 in Chapter 5 of the excellent book [164] of P.B. GILKEY. We have benefited greatly not only from this book, but also from the marvelous work [270] by H.B. LAWSON and M.L. MICHELSON. In that book, there are proofs of index formulas in various contexts, and numerous beautiful applications illustrating the power of Dirac operators, Clifford algebras and spinors in the geometrical analysis of manifolds, immersions, vector fields, and much more. The classical book [386] of P. SHANAHAN is also a masterful, elegant exposition of not only the standard index theorem, but also the G -index theorem and its numerous applications. A fundamental source on index theory for certain open manifolds and manifolds with boundary is the authoritative book [289] of R.B. MELROSE. In [363], TH. SCHICK reviews coarse index theory, in particular, for complete partitioned manifolds. It has been introduced by J. ROE and provides a theory to use tools from C^* -algebras to get information about the geometry of non-compact manifolds via index theory of Dirac type operators. [201] of N. HIGSON and J. ROE gives a well-written presentation of the underlying ideas of analytic K-homology and develops some of its applications. For a concrete calculation see also the concise [306, Section 7.4.2] and the Notes (forthcoming) [202]. See

also [134, 135, 136] of J. EICHHORN for heat kernel asymptotics on non-compact manifolds and [313] of B.-W. SCHULZE and collaborators for index theory on singular spaces. [454] of W. ZHANG gives an excellent introduction to various aspects of Atiyah-Singer index theory via the Bismut/Witten-type deformations of elliptic operators. Very close to our own view upon index theory is the plea [154] of M. FURUTA for reconsidering the index theorem, with emphasis on the localization theorem. In the case of boundary-value problems for Dirac operators, we put quite some care in the writing of our [80] jointly with K.P. WOJCIECHOWSKI. The recent book [152] of D. FURSAEV and D. VASSILEVICH contains a detailed description of main spectral functions and methods of their calculation with emphasis on heat kernel asymptotics and their application in various branches of modern physics. Following up on the classic [210] of F. HIRZEBRUCH and D. ZAGIER on interrelations between the index theorem and elementary number theory, the comprehensive [373] of S. SCOTT covers the theory of traces and determinants on Banach algebras of operators on vector bundles over closed manifolds, with emphasis on various algebras of pseudo-differential operators. He gives a series of calculations that give the flavor of the subject in tractable cases, and relates these calculations to Poisson and Selberg trace formulas. There is an impressive nonstandard proof of the local Atiyah-Singer index theorem, using resolvent expansions in place of the usual heat equation techniques. A wealth of radically new ideas of (partly yet unproven) geometric use of instantons are given in [266] of P.B. KRONHEIMER and T.S. MROWKA. Very inspiring is [223] of E.P. HSU on stochastic analysis on manifolds. It gives a reformulation of the heat equation proof of the index theorem in terms of Wiener process asymptotics. Basically, that is what we should have after A. EINSTEIN's famous 1905-discovery of the basis of heat conduction in diffusion. The details are interesting, though, in particular because they open a window to discrete analysis. A taste of the recent revival of D-branes and other exotic instantons in string theory can be gained from [161] of H. GHORBANI, D. MUSSO and A. LERDA. Indications can be found in the review [358] of F. SANNINO about, how strongly coupled theories of gauge theoretic physics result in perceiving a composite universe and other new physics awaiting to be discovered. In the mathematically rigorous and richly illustrated [352], N. RESHETIKHIN explains why and how topological invariants by necessity appear in various quantizations of gauge theories.

The Question of Originality: Seeking a Balance between Mathematical Heritage and Innovation. Parts I-III and the two appendices teach what mathematicians today consider general knowledge about the index theorem as one of the great achievements of 20th century mathematics. But, actually, there are two novelties included which even not all experts may be aware of: The *first novelty* appears when rounding up our comprehensive presentation of the topology of the space of Fredholm operators: we do not halt with the Atiyah-Jänich Theorem and the construction of the index bundle, but also confront the student with a thorough presentation of the various definitions of determinant line bundles. This is to remind the student that index theory is not a more or less closed collection of results but a philosophy of regularization, of deformation invariance and of visionary cross connections within mathematics and between its various branches.

A *second novelty* in the first three Parts is the emphasis on global constructions, e.g., in introducing and using the concept of pseudo-differential operators.

Apart from these two innovations, the student can feel protected in the first three Parts against any originality.

Basically, Part IV follows the same line. Happily we could also avoid excessive originality in the chapters dealing with instantons and the Donaldson-Kronheimer-Seiberg-Witten results about the geometry of moduli spaces of connections. There we also summarize, refer, define, explain great lines and details like in the first three Parts, though emphasizing variational aspects based on [59].

However, the *core* of Part IV is different. It consists of an original, full, quite lengthy (in parts almost unbearably meticulous) proof of the *Local Index Theorem for twisted Dirac operators* in Chapter 17 and its applications to standard geometric operators. That long Chapter is thought as a new contribution to the ongoing search for a deeper understanding of the index theorem and the “best” approach to it.

Clearly, a student looking for the most general formulation of the index theorem and a proof apt for wide generalizations should concentrate on our Part III, the so-called Embedding (or K-theoretic) Proof. However, a student wanting to trace the germs of index calculations back in the geometry of the considered standard operators (all arising from various decompositions of the algebra of exterior differential forms) should consult Section 17.5 with a full proof of the *Local Index Formula* for twisted Dirac operators on spin manifolds (all terms will be explained) and Section 17.6, where we derive the *Index Theorem for Standard Geometric Operators*. These geometric index theorems are by far less general than Part III’s embedding proof, but they are more geometric, and we hold, also more geometric than the *usual* heat equation proofs of the index theorem. Not striving for greatest generality, we obtain index formulas for the standard elliptic geometric operators and their twists. The standard elliptic geometric operators include the signature operator $d + \delta: (1 + *)\Omega^*(M) = \Omega^+(M) \rightarrow \Omega^-(M) = (1 - *)\Omega^*(M)$, the Euler-Dirac operator $d + \delta: \Omega^{\text{ev}}(M) \rightarrow \Omega^{\text{odd}}(M)$, and the Dolbeault-Dirac operator $\sqrt{2}(\bar{\partial} + \bar{\partial}^*): \Omega^{-,\text{ev}}(M) \rightarrow \Omega^{+,\text{odd}}(M)$ (all symbols will be defined). The index formula obtained for the above operators yields the Hirzebruch Signature Theorem, the Chern-Gauss-Bonnet Theorem, and the Hirzebruch-Riemann-Roch Theorem, respectively. While these operators generally are not globally twisted Dirac operators, locally they are expressible in terms of chiral halves of twisted Dirac operators. That applies also to the Yang-Mills operator. Thus, even if the underlying Riemannian manifold M (assumed to be oriented and of even dimension) does not admit a spin structure, we may still use the Local Index Theorem for twisted Dirac operators to compute the index density and hence the index of these operators. While it is possible to carry this out separately for each of the geometric operators, basically all of these theorems are consequences of one single index theorem for generalized Dirac operators on Clifford module bundles (all to be defined). Using the Local Index Theorem for twisted Dirac operators, we prove this index theorem first (our Theorem 17.59), and then we apply it to obtain the geometric index theorems, yielding the general Atiyah-Singer Index Theorem for practically all geometrically defined operators.

Style and Notations. To present the rich world of index theory, we have chosen two different styles. We write all definitions, theorems, and proofs as concise as possible to free the reader from dispensable side information. Where possible, we begin the introduction of a new concept with a simple but generic example or a review of the local theory, immediately followed by the corresponding global or

general concept. That is one half of the book, so to speak the odd numbered pages. The other half of the book consists of exercises (often with extended hints) and historical reviews, motivations, perspectives, examples. We wrote those sections in a more open web-like style. Important definitions, notions, concepts are in bold face. Background information is in small between the signs ► and ◀. In remarks and notes, leading terms are in italics.

The reader will notice our bias towards elder literature when more recent references would not add substantially more value. This is due not so much to the age of the authors (both born before the middle of the last century) but rather to the common pride of mathematicians belonging to a community where bibliographic impact factors and research indices should rather be calculated in citations after some decades of years than in numbers of recently appeared, cited and soon forgotten publications.

There is also a distinction, due to HARALD BOHR and disseminated by BØRGE JESSEN, between *expansive* and *consolidating* periods of each individual science. While physics and biology had consolidating periods in the first half of the last century and suffer now of the rapid change of ever new single and dispersed results, mathematics has had and still has good decades of consolidation and of long-time validity of key results. To the present authors, there is no reason to hide our indifference to changing fashions.

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Both authors agreed to dedicate this book to their teachers, to the memory of S.-S. CHERN (thesis adviser of DB) on the occasion of his centenary in October 2011 and to the memory of F. HIRZEBRUCH (thesis adviser of BBB) who appreciated the announced dedication intended for his 85th birthday in October 2012, which he did not live to celebrate. I take the liberty to change the dedication. This book is dedicated to DAVID.

Bernhelm Booß-Bavnbek, Roskilde (Denmark), December 2012

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Index of Notation

- Δ Laplace operator
- Δ_P generalized Laplace operator, 305
- Beltrami Laplacian, 189
- connection (covariant) Laplacian on $\Omega^k(W)$, 439
- connection Laplacian, 199, 536
- Dirac Laplacian, 187
- Euclidean, 144
- Hodge Laplacian, 439
- $\Gamma(s)$ Gamma function, 112, 204
- Γ_{jk}^i Christoffel symbols
 - for connection, 178, 421
 - for Riemannian metric, 168
- $\Lambda^\bullet(V)$ graded exterior algebra of V , 171, 514
- $\Lambda_{E,s}$ generating operator for Sobolev spaces, 198
- $\Lambda^\bullet(T^*X)$ total bundle of exterior forms, 172
- $\Lambda^p(T^*X)$ vector bundle of exterior forms, 172
- $\Lambda^p(V)$ vector space of p -fold skew-symmetric tensors, 171
- Λ_s generating operator for Sobolev spaces, 197
- $\Omega^k(P, W)$ smooth equivariant W -valued k -forms on P , 401
- $\Omega^{0,k}$ space of complex forms of type $(0,k)$, 187
- $\Omega^{p,q}(M)$ forms of bidegree (p, q) , 616
- $\overline{\Omega}^k(P, W)$ smooth horizontal equivariant W -valued k -forms on P , 401
- Ω^ω curvature of connection ω , 399
- $\Omega^\bullet(X)$ total space of exterior differential forms, 172
- $\Omega^p(X)$ space of exterior differential p -forms, 172
- $\Omega_{\pm}^2(M, \mathbb{R})$ spaces of (anti-) self-dual forms, 405
- Φ Thom isomorphism of singular cohomology, 311, 313
- Φ^\times Seiberg-Witten function, 665
- Φ isomorphism from horizontal equivariant forms to the gauge group, 411
- Ψ Thom isomorphism of K -theory, 290, 313
- Ψ inverse of Φ , 411
- ${}_{,}\Sigma(M), \Sigma^\pm(M)$ the Hermitian spinor bundles, 531
- $\Sigma_c^\pm(X), \Sigma_c(X)$ bundles associated to Spin^c structure, 659
- Σ_{2m} vector space of spinors, 521
- Σ_{2m}^\pm vector space of half-spinors, 521
- $\Sigma_{c,g}(X), \Sigma_{c,g}^\pm$ bundles of virtual twisted spinors relative to g , 696
- Σ_{v_1, \dots, v_r} singular set of array of vector fields, 325
- Θ torsion of a connection 1-form, 415
- $\alpha: K(\mathbb{R}^2 \times X) \xrightarrow{\sim} K(X)$ Bott isomorphism, 270
- $\beta(T)$ symplectic space of all extensions of closed symmetric T , 49
- $\beta(A, B)$ Killing form, 462
- χ Euler characteristic / class of a surface, 117
 - complex, 7
 - complex vector bundle, 313
 - real Riemannian bundle, 453
 - topological manifold, 262, 320, 609
- $\chi: U(1) \times \text{Spin}(2m) \rightarrow \text{Spin}^c(2m)$ 2-fold cover, 659
- χ_{hol} holomorphic Euler characteristic of a complex manifold = arithmetic genus, 334
 - of a holomorphic vector bundle, 334, 642
- δ
 - Bockstein homomorphism of homological algebra, 655
 - codifferential operator on forms, 188
 - difference bundle construction, 263
 - Dirac distribution, 38, 136
- ∂X boundary of a manifold X , 143
- δ^ω covariant codifferential, 406
- ε -tensor, 96
- $\eta_i, \overline{\eta}_i$ 't Hooft matrices, 467
- $\eta_D(s)$ eta function of Dirac type operator D , 111
- $\tilde{\eta}_D$ reduced eta invariant, 112
- γ gap (projection) metric for $\mathcal{C}(H)$, 52
- γ_5 global section, “ γ_5 -matrix”, 340
- κ general heat kernel, 547
- $\kappa(z)$ Cayley transformation, 130
- $\kappa \in \Omega^{1,1}(M)$ Kähler 2-form, 618

- λ size of instanton, 475
 λ_V canonical difference element of $\Lambda^\bullet(V)$
 for complex vector bundle $V \rightarrow X$, 289
 $\mu(P)$ boundary index of an elliptic
 operator, 249
 $\nu, \frac{\partial}{\partial \nu}, \mathbf{n}$ field normal to the boundary, 154,
 181, 191
 ν_C normalized γ_5 -matrix, 659
 ν_g volume form of X with metric tensor g ,
 170, 405
 $[\omega, \omega]$ bracket of \mathfrak{g} -valued 1-form, 399
 ω connection 1-form on P , 398
 ω symplectic form on smooth manifold, 652
 ω_C complex volume element, 520
 ϕ_\times associated vector bundle isomorphism,
 409
 $\varphi(a)$ autocorrelation function, 124
 $\pi: P \rightarrow M$ principal G -bundle, 395
 $\pi|_{P_0}: P_0 \rightarrow M$ holonomy bundle, 456
 $\pi_k(X, x_0)$ homotopy group
 $\pi_1(X, x_0)$ fundamental group, 77, 128
 $\pi_{r,c}$ r^c -equivariant bundle map defining a
 $\text{Spin}^c(n)$ structure, 655
 π_E vector bundle base point projection, 181
 ρ, ρ^\pm, ρ_C spinor representations, 519–521
 $\rho(X)$ injectivity radius of Riemannian
 manifold X , 169
 $[\sigma(P)]$ symbol class, 279, 289
 $\sigma: \mathcal{B} \rightarrow U(E) \times_f P$ radial gauge for fixed
 $x \in M$, 552
 σ_i Pauli matrices, 467
 $\sigma(P)$ principal symbol bundle
 homomorphism of
 $P \in L_{\text{pc}}^k(E, F)$, 214, 228
 differential operator, 184
 $\sigma(P)(x, \xi_x)$ principal symbol
 of $P \in L_{\text{pc}}^\bullet(E, F)$, 214, 217, 228
 of Bokobza-Haggiag amplitude, 234
 of differential operator, 183, 216
 $\sigma_k(\lambda_1, \dots, \lambda_\nu)$ elementary symmetric
 polynomial of degree k in x_1, \dots, x_ν ,
 448
 τ involution on forms, 173
 $\tau_{c,t}^E$ parallel translation
 along path c , 178
 $\tau_{x,x'}^E$ parallel translation
 within injectivity radius, 179
 τ_E, τ_F local bundle trivialization, 183
 (θ_j^i) local Levi-Civita connection forms,
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 θ Levi-Civita connection, 180, 418
 θ_\pm decomposition of Levi-Civita
 connection, 467
 φ canonical 1-form on LM , 415
 $\zeta_P(s)$ zeta function of semi-positive elliptic
 operator P , 114
 $|A|$ symmetric (absolute) factor of operator
 A , 56
 \mathcal{A} atlas of charts, 157
 \mathcal{A}^\times group of units of Banach algebra \mathcal{A} , 64,
 65
 \mathcal{A} space of connections (affine
 configurations space), see $\mathcal{C}(P)$, 461
 $\tilde{A}(M)$ generalized total characteristic class,
 598
 A^* fundamental vertical vector field of
 element A in Lie algebra, 398
 ad adjoint representation of G on \mathfrak{g} , 397
 Ad_g adjoint action of G on G , 397
 ad derivative of ad at identity, 397
 \hat{A} A roof
 $\hat{A}(F)$ total \hat{A} class of real Riemannian
 bundle $F \rightarrow M$, 451
 $\hat{A}(M) := \hat{A}(TM)$ total \hat{A} class of
 oriented Riemannian manifold M
 with spin structure, 546
 A_f transmission (coupling) operator, 272
 $\text{Ampl}_k(E, F)$ space of amplitudes of order
 k , 234
 ant antipodal map
 on sphere, 274
 on tangent bundle, 317
 A^\pm pull-backs of θ_\pm to \mathbb{R}^4 , 467
 $\mathcal{A}_{\mathcal{R}}$ closed extension (realization) of
 operator with domain \mathcal{R} , 343
 $\text{Aut}(P)$ group of all automorphisms of P ,
 409
 B real elliptic operator associated to Spin^c
 structure, 666
 $B^2(\mathcal{U}; \mathbb{Z}_2)$ group of Čech 2-coboundaries
 with values in \mathbb{Z}_2 relative to the cover
 \mathcal{U} , 527
 B, B^\pm, B, \not{B} tangential Dirac operators, 341
 \mathcal{B} quotient space (space of moduli of all
 connections), see $\mathcal{M}(P)$, 461
 \mathbf{b} Bott class, 270
 b equivariant K -theory element, 292
 b_k Betti number, 7
 $B(H)$ Banach algebra of bounded operators
 on a Hilbert space H , 3
 B^+ convex set of positive operators, 17
 b^+ self-dual Betti number, 653
 B_f ideal of finite rank operators, 56
 B_H closed unit ball in Hilbert space, 18
 $C^2(\mathcal{U}; \mathbb{Z}_2)$ group of Čech 2-cochains with
 values in \mathbb{Z}_2 relative to the cover \mathcal{U} ,
 526
 $C_\downarrow^\infty(\mathbb{R})$ Schwartz space of rapidly
 decreasing functions, 113
 \mathbb{C}^\times complex units, 253
 $\mathcal{C}_j, j = 1, \dots, 4$ $\text{SO}(4)$ -irreducible
 decomposition for $n = 4$, 432

- \mathcal{C} : $P \rightarrow FM$ spin structure for Riemannian manifold M , 525
- $\dot{c}(0)$ tangent vector of a path c , 160
- \mathbf{c} (left) Clifford multiplication, 180, 340, 531, 659
- $c(E)$ total Chern class of complex vector bundle E , 312
- c : $\text{Spin}(n) \rightarrow \text{SO}(n)$ double covering homomorphism, 517
- $c_k(E)$ Chern class of complex vector bundle E , 312, 444
- $C(P, G)$ smooth equivariant G -valued functions on P , 410
- $C^0(S^1)$ Banach space of continuous \mathbb{C} -valued functions on S^1 , 705
- $c_1: H^1(X; \mathbb{U}(1)) \rightarrow H^2(X; \mathbb{Z})$ Čech cohomology isomorphism, 656
- $C^\infty(\mathbb{R}^n)$ complex valued C^∞ function on \mathbb{R}^n , 135
- $C^\infty(TX)$ space of smooth vector fields, 165
- $\mathcal{C}(H)$ space of closed densely defined operators in H , 52
- $\text{ch}(E)$ Chern character, 260, 312, 445
- $\mathcal{CF}(H)$ space of closed (not necessarily bounded) Fredholm operators, 52
- $C^\infty(\mathbb{R}^n, \mathbb{C}^N)$ C^∞ functions from \mathbb{R}^n to \mathbb{C}^N , 139
- $C^\infty(X; E)$ linear space of smooth sections of E , $= C^\infty(E)$, 181
- $\text{Cl}(TX)$ complex Clifford bundle, 595
- $\text{Cl}(X)$ Clifford bundle on Riemannian manifold, 180
- $\text{Cl}^\pm(X)$ chiral Clifford bundles, 181
- \mathbf{c}_g Clifford multiplication relative to g , 697
- $\text{Cl}(TX_x, g_x)$ Clifford algebras of tangent vectors, 180
- $\text{Cl}(V)$ Clifford algebra of real vector space V , 514
- Cl_{2m} complex Clifford algebra, 520
- Cl_n Clifford algebra of real n space, 514
- C_0^∞ smooth sections with compact support, 186
- $\text{Coker } T$ cokernel of the operator T , 3
- $\mathcal{C}(P)$ space of C^∞ connections on P , 396
- $\mathcal{C}(P)^+$
 - space of self-dual connections, 489
 - space of self-dual weakly irreducible connections over fixed 4-manifold, 496
- $\mathcal{C}(P)^{p,k}$ Sobolev space of connections, 500
- $\mathcal{C}(P)_m^+$ mildly-irreducible self-dual connections, 496, 507
- $\mathbb{C}P^2$ complex projective space of complex dimension 2, 647
- $\mathcal{CW}\mathcal{P}_k$ Banach manifold of connections, parametrized by Sobolev space, 668
- \mathbb{C}_X^N trivial product bundle of complex fiber dimension N over base space X , 181
- $C(Y)$ semi-group of equivalence classes of complexes of vector bundles with compact support, 277
- $C(Y, X)$ space of continuous mappings from Y to X , 18
- $D^{\omega \oplus \theta}$ exterior derivative on $P \times_G W$ -valued k -forms, 435
- \mathcal{D} Dirac operator
 - \mathcal{D}^\pm (partial) chiral Dirac operators, 181, 532
 - \mathcal{D}_c Spin^c -Dirac operator, 659
 - $\mathcal{D}_c^{(\omega, L_\varphi^* \theta_2)}$ lifted Spin^c -Dirac operator, 697
 - \mathcal{D} (free) Euclidean Dirac operator, 341
 - \mathcal{D}_Λ twisted Euclidean Dirac operator, 342
 - operator of Dirac type, 180
 - standard Dirac operator for spin structure, 531
 - twisted Dirac operator, 532
- $\mathcal{D}^W, \mathcal{D}^{W^\pm}$ Dirac operator
 - generalized Dirac operators, 597
- $\mathcal{D}^{W^{\text{odd}}, E}, \mathcal{D}^{W^{\text{ev}}, E}$ Dirac operator
 - Yang-Mills Dirac operators, 613
- $\mathcal{D}^{W^{\text{odd}}}, \mathcal{D}^{W^{\text{ev}}}$ Dirac operator
 - generalized Dirac operators, 608
- \mathcal{D} divisor on Riemann surface, 335
- $(d + \delta)^+$ signature operator, 321, 604
- $(d + \delta)^{E,+}$ twisted signature operator, 346, 606
- $(d + \delta)^{\text{ev}}, (d + \delta)^\times$ Euler operator, 320, 609
- $\frac{\partial}{\partial \bar{z}}$ Cauchy-Riemann operator, 187
- $\bar{\partial}$ Dolbeault operator, 187, 334
- $\bar{\partial}_E$ generalized Dolbeault operator, 334, 639
- $\partial_{x_1}^2 + \dots + \partial_{x_n}^2$ Euclidean Laplace operator, 187
- deg_c degree of contribution, 579
- d exterior differentiation, 172
- $d + \delta$ deRham-Dirac operator, 320, 602
- $d_k, d_{\bar{k}}$ virtual dimensions of moduli spaces $M_k, M_{\bar{k}}$ of self-dual and anti-self-dual connections for classes of principal $\text{SU}(2)$ -bundles, 650
- D^α (symmetrized) partial derivative of multiindex α , 135, 182
- D^ω covariant derivative relative to ω , 398
- $d(E^\bullet), \chi(E_0, E_1; \sigma)$ difference bundle for complex E^\bullet , 278, 288
- deg degree
 - $\text{deg}(P)$ local index of an elliptic operator, 249
 - $\text{deg } \mathcal{M}^+$ boundary clutching degree of an elliptic operator, 249
 - $\text{deg}(\mathcal{D})$ degree of divisor \mathcal{D} , 335

- $\deg(f)$ winding number, mapping degree, 254, 257–259
 det determinant
 $\pi: \mathcal{S} \rightarrow \mathcal{F}_0$ Segal determinant line bundle, 102
 ζ -regularized determinant, 119
 $\det \alpha$ exterior determinant of $\alpha \in K(X)$, 92
 $\det F$ determinant line of Fredholm operator F , 693
 $\det T$ exterior determinant of family of Fredholm operators, 92
 Fredholm determinant, 97, 98
 $q: \mathcal{Q} \rightarrow \mathcal{F}$ Quillen determinant line bundle, 95
 $\text{Diff}_k(E, F)$ space of linear differential operators of order $\leq k$, 182
 d_∞ uniform metric, 18
 dist distance, 168
 div divergence of a vector field, 185
 $\text{Dom}(T)$ domain of (not necessarily bounded) operator T , 35

 $E P \times_G \mathfrak{g}_c$ and/or all-purpose bundle, 491
 E spectral measure, 51
 \mathring{E} dotted vector bundle, i.e., after removing the zero-section, 182
 $\mathcal{E}(r, t)$ fundamental solution of standard heat equation in Euclidean n -space, 550, 563
 $E(c)$ energy functional, 168
 E_8 unimodular rank 8 even quadratic form of signature 8 (*exceptional form*), 325, 646
 \mathcal{E} (exceptional) lattice in \mathbb{R}^{4k} , 647
 \mathfrak{e}_8 exceptional Lie algebra, 647
 $\text{Ell}_{\text{Bokobza}}^{O(m), k}(E, F)$ space of $O(m)$ -invariant elliptic amplitudes of order k , 295
 $\text{Ell}_{\text{Bokobza}}^k(E, F)$ space of elliptic amplitudes of order k , 235
 $\text{Ell}(X)$ class of elliptic pseudo-differential operators on closed manifold X , 265
 $\text{Ell}_c(\mathbb{R}^n)$ class of elliptic pseudo-differential operators of order 0 in \mathbb{R}^n being $= \text{Id}$ at ∞ , 275
 $\text{Ell}_k(E, F)$ space of elliptic principally classical pseudo-differential operators of order k , 239
 E_m m -times twisted line bundle defined over S^2 , 270
 $\text{End}(V)$ algebra of \mathbb{C} -linear endomorphisms of complex vector space V , 519
 exp exponential map
 $\exp(A)$ of matrix A in Lie algebra, 397
 $\exp: \mathcal{Cl}(V) \rightarrow \mathcal{Cl}(V)$, 514
 \exp_x of Riemannian manifold X at x , 169
 $\text{Exp}: C(P, \mathfrak{g}) \rightarrow C(P, G)$, 413

 Exp_s of Sobolev Banach manifolds at $s \in W^{2, k+2}(X, \mathbb{C})$, 669
 ext natural extension in topological K -theory, 285

 \mathcal{F} correction operator between usual and connection Laplacian, 564
 \hat{f} Fourier coefficient, integral, transform, 706, 710
 $f_!$: $K(TX) \rightarrow K(TN)$ induced homomorphism of smooth proper embedding $f: X \hookrightarrow Y$ with normal bundle N , 290
 $f_*|_x$ differential of $f: X \rightarrow Y \in C^\infty$ at $x \in X$, 161
 $\mathcal{F}(H)$ space of Fredholm operators on a Hilbert space H , 3
 \mathcal{F}_0 set of Fredholm operators of index 0, 88
 f^* lifting (pull-back)
 of differential forms, 165
 of vector bundles, 713
 $F \cdot \alpha$ action of $\text{GA}(P)$ on $\mathcal{C}(P)$ or $\overline{\Omega}^k(P, W)$, 410
 $F_i(X) = H_i(X; \mathbb{Z})/T^i(X)$ essential (torsion-free) homology, 658
 $\pi: FM \rightarrow M$ orthonormal frame bundle, 416
 F^\pm field strengths of A^\pm , 467
 $(\mathbf{FR}_k)^c$ set of unsuitable pairs, 700
 \mathcal{F}_T affine Fredholm version of \mathcal{I}_1 , 104
 $FX(g)$ bundle of oriented orthonormal frames for the metric g , 694

 G Lie group of matrices, 394
 $\mathfrak{G}(T)$ graph of an operator T , 36
 \mathcal{G} gauge group, see $\text{GA}(P)$, 461
 \mathfrak{g} Lie algebra of G , 397
 GA Grothendieck group of abelian semi-group, 262
 $\text{GA}_1(P_{\text{Spin}^c(n)})$ subgroup acting on the solution space of perturbed S-W equations, 663
 $\text{GA}(P)$ group of C^∞ gauge transformations of P , 409
 $\text{GB}(\Omega^\theta)$ Gauss-Bonnet form of a connection θ with curvature form Ω , 321, 446
 $\mathfrak{g}_\mathbb{C}$ complexification of \mathfrak{g} , 464
 $g = (g_{ij}(x))$ metric tensor, 167
 $\text{GL}(N, \mathbb{C})$ Lie group of invertible complex $N \times N$ matrices, 42
 $\text{gl}(N, \mathbb{C})$ Lie algebra of complex $N \times N$ matrices, 41
 $\text{Grass}^{\text{sa}}(\mathcal{D})$ self-adjoint Grassmannian, 348
 Grass_{p+} Grassmannian of pseudo-differential projections with the same principal symbol, 345

- $g(S)$ genus (Klassenzahl) of surface $S =$
 numerical complexity of Abelian
 integral, 319, 332
- H hyperbolic intersection form, 646
- $H^k(\mathcal{U}; \mathbb{Z}_2)$ Čech cohomology group with
 values in \mathbb{Z}_2 relative to the cover \mathcal{U} ,
 527
- $H^q(M)$ deRham cohomology space, 603
- $H^q(\mathcal{O}_E)$ generalized Dolbeault cohomology
 groups, 334, 639
- $H^{0,q}(M)$ Dolbeault cohomology space, 617
- $H^*(X; R)$ cohomology functor with
 coefficients in ring R , 266
- $H_Q(x, y, t), G_Q(x, y, t)$ approximative heat
 kernels, 566, 567
- $H_c^*(\cdot)$ cohomology with compact supports,
 312
- H_{\pm} discrete Hardy spaces, 125
- \mathbb{H} algebra of quaternions, 514
- $\mathcal{H}(\Omega^\omega)$ harmonic part of Ω^ω , 691
- $\mathcal{H}^k(M)$ space of harmonic k -forms, 602
- \mathcal{H}^+ L^2 -orthogonal projection onto the
 self-dual harmonic 2-forms relative to
 g , 691
- \hbar reduced Planck constant, 190
- \mathcal{H} (discrete) Hilbert transform, 127
- $\mathcal{H}_+(A)$ Cauchy data space, 344
- $H_k(C)$ homology space of a complex C , 7
- H_N^+ convex space of positive-definite
 Hermitian matrices, 74
- $\text{Hol}(\omega, p_0)$ holonomy group, 455
- $\text{Hom}(E, F), \text{Iso}(E, F)$ bundles of vector
 bundle homomorphisms and
 isomorphisms, 713, 715
- $\text{Hom}_0(\Sigma_{2m}, V)$ space of Cl_{2m} -equivariant
 linear maps, 523
- H_p, V_p horizontal, vertical subspace of $T_p P$
 at $p \in P$, 396
- H_V Hopf bundle, 714
- $I(P_0)$ index of a singular point, 252
- $I(p, \varphi)(x)$ oscillatory integral, 218
- I_ω gauge isotropy subgroup at ω , 456
- \Im imaginary part, 331
- \mathcal{I}_1 ideal of trace class operators, 56
- \mathcal{I}_2 ideal of Hilbert–Schmidt operators, 56
- \mathcal{I}_p Schatten class, 56
- Id identity operator, 9
- $\mathcal{I}(E)$ Chern character defect, 314
- $\text{Im}(T)$ image of an operator T , 3
- I_n $n \times n$ unit matrix, 165
- index
 - $\text{index}(M, N)$ index of a Fredholm pair
 (M, N) of closed subspaces, 344
 - index_t topological index, 287
 - $\text{index}_{\text{O}(m)}$ $v \text{O}(m)$ character of
 equivariant K -class v , 294
- index
 - $\text{index}_G P$ (analytic) index of an elliptic
 G -operator, 360
 - $\text{index}_g P$ (analytic) virtual character, 360
 - $\text{index}_{t,G} P$ topological G -index, 361
 - $\text{index}_{t,g} P$ topological g -index, 361
 - index_a analytic index, 286, 291
 - index_t topological index, 286, 290
 - index bundle of a continuous family of
 Fredholm operators, 84
 - index of a continuous family of Fredholm
 operators, 82
 - index of a Fredholm operator, 3
 - $\mathbf{i}(P_2, P_1)$ virtual codimension of a Fredholm
 pair of projections, 345
 - $I(X)$ group of stable equivalence classes of
 bundles, 264
- $J(= G)$ Green's function on boundary, 181
- J_ε mollifier, 196
- \mathcal{J} integration operator, 38
- $\mathcal{J}(M, \omega)$ set of complex structures
 compatible with symplectic form ω ,
 652
- $J^k(E)$ k -jet bundle, 166
- $j_k(f)_x$ k -jet of section f at x , 166
- $J_x^k(E)$ k -jets of complex vector bundle E at
 x , 166
- $K(x, y)$ weight (kernel) of integral
 operators, 212
- $K(A, B)$ ad -invariant inner product on \mathfrak{g} ,
 461
- $K(\Pi)$ Gaussian curvature, 423
- $K_{Q,0}(x, y, t)$ heat kernel error term, 567
- \mathcal{K} ideal of compact operators, 18
- k, k^\pm twisted spinorial heat kernels, 541
- $K_{\mathbb{C}}$ complexified Killing type form, 464
- $\text{Ker } T$ kernel of an operator T , 3
- $K_G(X)$ equivariant K -group, 293
- $K_{\text{O}(m)}(T^*S^m)$ equivariant K -group, 294
- K_φ convolution operator, 129
- $K(X)$ K -group of topological space X , 83,
 263, 266
- $K(X, Y)$ relative K -group, 264, 266
- K_S canonical divisor of Riemann surface S ,
 336
- K_X canonical class of compatible almost
 complex structure, 653
- L canonical line bundle on Spin^c -manifold,
 659
- $L^\bullet(E, F)$ space of canonical
 pseudo-differential operators, 210
- $L_{\text{Bokobza}}^\bullet(E, F)$ space of Bokobza-Haggiag
 pseudo-differential operators, 234
- $L_{\text{pc}}^\bullet(E, F)$ space of principally classical
 pseudo-differential operators, 214, 216,
 224, 225, 229

- L_φ equivariant transformation of oriented orthonormal frames, 695
 $L_q(p_1, \dots, p_q)$ Hirzebruch L -polynomials, 324
 $\mathbf{L}(F)$ total Hirzebruch L class, 451
 \mathcal{L} linear isomorphism $\Lambda^\bullet(V) \rightarrow C\ell(V)$, 514
 $L(c)$ length functional, 168
 $L^1(S^1)$ Banach space of \mathbb{C} -valued integrable functions on S^1 , 705
 $L^2(E) = L^2(X; E)$ Hilbert space of Lebesgue measurable square integrable sections in a Hermitian vector bundle E on a Riemannian manifold X , 194
 $L^2(S^1)$ Hilbert space of square-integrable \mathbb{C} -valued functions on S^1 , 705
 $L^2(X)$ Hilbert space of complex valued square integrable functions on X , 10
 $L^2(\mathbb{Z}) = \ell^2$ Hilbert space of square summable series, 4
 $L_{\text{sym}}^k(V, W)$ space of symmetric k -linear forms on V with values in W , 166
 $\mathcal{L}_A B = [A, B]$ Lie bracket of vector fields A, B , 174
 $L(f)$ Lefschetz number, 357
 $L(f, P)$ Atiyah-Bott-Lefschetz number, 358
 L_g, ℓ_g left action of g , 292, 360, 397
 $LM \rightarrow M$ bundle of linear frames, 414
 $L^p(E)$ L^p sections of $E \rightarrow M$, 497

 M_η moduli space for Spin^c structure, 664
 $[M]$ fundamental cycle of oriented manifold M , 313
 \mathcal{M} space of moduli of connections, 489
 \mathcal{M}^+ moduli of $\mathcal{C}(P)_m^+$, 510
 \mathcal{M}^\pm eigenspaces of boundary symbol, 245
 \mathcal{M} space of moduli of self-dual connections, see $\mathcal{M}^+(P)$, 461
 m mass of a particle, 190
 $m: \mathbb{Z} \rightarrow \mathbb{Z}$ multiplication by 2, 655
 \mathbf{MC} multiplicative class assigned to a characteristic class, 449
 \mathcal{MCWP}_k quotient space of moduli, 673
 $\mathcal{MCWPS}_k, \mathcal{MSWPS}_k$ Hausdorff C^∞ Hilbert (quotient) manifolds, 698
 $\text{Met}(M)$ convex set of metric tensors, 652
 \mathbf{MF} multiplicative form, 449
 M_{id} spectral multiplication operator, 36
 $M_1 \# M_2$ connected sum, 645
 \mathcal{M}^+ space of moduli of self-dual connections, 489

 \odot Cayley numbers, 607
 \mathcal{O}, o Big Oh, Small Oh, 549
 \mathcal{O}_E generalized Dolbeault complex, 639
 $\text{OP}_k(E, F)$ space of operators of order k , 237
 $\text{Op}(p)$ pseudo-differential operator (quantization) of amplitude p
- Bokobza-Haggiag (global) construction, 231, 233, 234
 Euclidean construction, 210
 patched global construction, 229
 $\text{OP}_{p,1}$ Banach space of bounded operators $W^{p,k+1}(E) \rightarrow W^{p,k}(E \otimes \Lambda^1(X))$, 502

 P Poincaré duality, 644
 $P \times_f FM$ fibered product, 434
 P^\pm chiral splitting of an operator P , 47
 $\mathbb{P}V$ projective space of complex vector space V , 714
 $p(x, \xi)$ total symbol/amplitude (dequantization), 218
 $\mathbb{P}(TX)$ set of all tangent lines over all points on a manifold X , 164
 $P_{\geq}(B)$ spectral (Atiyah–Patodi–Singer) projection, 345
 $\mathcal{P}_+(A)$ Calderón projection, 344
 P^* formally adjoint operator, 184
 $\text{Pf}(A)$ Pfaffian of $A \in \mathfrak{so}(2m)$, 446
 $P \times_G W$ associated vector bundle, 400
 $p_k(\Omega^\theta)$ Pontryagin forms, 546
 $p_k(M)$ Pontryagin class, 447

 $[Q]$ matrix of quadratic form Q , 646
 $q: C^\infty(\Sigma_c^+(X)) \rightarrow \Omega^{2+}(X, i\mathbb{R})$ quadratic map, 661
 $q_{d_k}(X)$ Donaldson’s polynomial invariants, 650
 Q^X intersection form on $H^2(X; \mathbb{Z})$, 643

 $R(W, Z, X, Y)$ curvature tensor, 422
 \mathcal{R} vector space of curvature tensors, 427
 \mathcal{R}^ε transformation of twisted spinors, 537
 $\mathfrak{R}^\omega, \mathfrak{R}^\pm$ correction forms for Spin^c -Dirac-Laplacian, 660, 661
 $r: \mathcal{R}(\mathbb{R}^n) \rightarrow \mathcal{S}(\mathbb{R}^n)$ Ricci map, 427
 $r^c = Sq \times c: \text{Spin}^c(n) \rightarrow \text{U}(1) \times \text{SO}(n)$ double-covering homomorphism, 655
 $\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3$ $O(n)$ -irreducible decomposition of \mathcal{R} , 428
 R^α Riesz (singular integral) operator, 212
 $r_*: H^2(X; \mathbb{Z}) \rightarrow H^2(X; \mathbb{Z}_2)$ homology reduction mod 2, 649
 \Re real part, 20
 $\text{Res}(T)$ resolvent set of the operator T , 10, 50
 $R(\lambda)$ resolvent function, 50
 R_g right action of g on P , 396
 $R(H)$ representation (Grothendieck) ring of group H , 293, 360
 $\text{Ric}(X, Y)$ Ricci tensor, 426

 S scalar curvature, 426
 $S(Y)$ suspension of topological space Y , 718
 S_{pc}^\bullet principally classical symbols/amplitudes, 214

- S_ω slice of action through ω , 489
 S symmetric bilinear forms, 427
 $S(M)$ set of inequivalent spin structures on manifold M , 528
 $S^\bullet(U \times \mathbb{R}^n)$ symbols of Hörmander type (1, 0), 210
 $s \cdot \omega$ infinitesimal action of $C(P, \mathfrak{g})$ on $\mathcal{C}(P)$, 413
 $s: \mathcal{R}(\mathbb{R}^n) \rightarrow \mathbb{R}$ scalar map, 427
 $S_{g_1}^+$ X bundle of positive g_1 -symmetric operators, 694
 $\text{sgn}(\sigma)$ sign of permutation σ , 171
 shift^\pm shift operators, 4
 sig signature
 of a quadratic form Q , 320, 646
 of a topological manifold X , 320
 s_k homogeneous polynomial of degree k , 443
 $s_k(\Omega^\omega)$ Chern form, 444
 $\text{Smb}l_k(E, F)$ space of k -homogeneous bundle homomorphisms on \hat{T}^*X , 184, 225
 $\text{SO}(n)$ special orthogonal group, 396
 $\mathfrak{so}(n)$ Lie algebra of antisymmetric $n \times n$ matrices, 515
 $\text{Sol}(\eta)$ set of all C^∞ solutions of the $S - W$ equations perturbed by η , 692
 $\text{span} = [\dots]$ linear span of vectors, 21, 400
 Spec spectrum, 10, 50
 Spec_c continuous, 10
 Spec_e essential, 10
 Spec_p point (discrete), 10
 Spec_r residual spectrum, 10
 $\text{spec}(\mathcal{A}, \mathcal{A}^*)$ domain for spectral invariance of operators belonging to a set \mathcal{A} , 110
 $\text{Spin}(n)$ spin group, 515
 Spin^c
 $\text{Spin}^c(n)$ n th Spin^c -group, 655
 $\text{Spin}^c(n)$ structure for an oriented Riemannian n -manifold, 655
 $\mathfrak{spin}(n)$ spin algebra, 515
 Str supertrace
 of heat kernel k , 544
 of spinor endomorphism A , 522
 $\text{str}(W^s(X))$ strength of a Sobolev space, 201
 $\text{SU}(n)$ special unitary group, 395
 supp support, 136
 $\text{SW}: H^2(X; \mathbb{Z}) \rightarrow \mathbb{Z}$ Seiberg-Witten invariants (relative to one fixed Spin^c structure), 652
 $\text{SW}([P_{\text{Spin}^c}])$ Seiberg-Witten invariant, 693
 $\text{SW}_\infty(\eta)$ set of C^∞ solutions of the S-W equations modulo $C^\infty(X, U(1))$, 691
 $\text{SW}_k(\eta)$ smooth submanifold of \mathcal{MSWP}_k , 680
 SWP_k parametrized solution space, 669
 T operator, $\Omega^1(E) \rightarrow \Omega^0(E) \oplus \Omega_-^2(E)$, 491
 $T^i(X)$ torsion subgroup of $H^i(X; \mathbb{Z})$, 657
 \mathcal{T} algebra of discrete Wiener-Hopf operators, 132
 $T(X, Y)$ torsion tensor, 179
 T^2 2-dimensional torus, 142
 T^*X differential, $= \varphi_{*x}$, 164
 $\mathbf{Td}(E)$ Todd class of a complex vector bundle E , 312, 450
 T_f discrete Wiener-Hopf operator, 69, 125, 255
 T^n n -dimensional torus, 201
 $\text{Tr } A, \text{tr } A$ trace of $A \in \mathcal{I}_1$, 58
 $\text{Tr } R$ trace of curvature tensor R , 427
 $T^{r,s}$ $P \times_G W$ -valued tensors, 434
 $T_x X$ tangent space of X at x , 160

 $U(H)$ group of unitary operators, 73
 $U(N)$ group of unitary $N \times N$ matrices, 73

 V_λ eigenspace, 538
 V^* dual vector space, 171
 $\text{Vect}(X)$ abelian semi-group of isomorphism classes of complex vector bundles over X , 83, 262, 716
 $\text{vol}(X)$ volume of Riemannian manifold X , 170

 W Sobolev spaces
 W^k modeled on L^2 , 35
 $W^s(E)$ bundle sections, 197
 $W^s(\mathbb{R}^n)$ Euclidean, 195
 $W^m(\mathbb{R}_+^n)$ over half spaces, 200
 W_K^s compactly supported, 198
 $W^{p,k}$ modeled on L^p
 $W^{p,k}(E)$ bundle sections, 497
 $W(M)$ Clifford module bundle, 595
 $W(f, 0)$ winding number
 see $\text{deg}(f)$, 69, 125
 W^\pm (anti-) self-dual part of \mathcal{R}_3 , 433
 $w_2(M)$ second Stiefel-Whitney class of M , 527
 W_f continuous Wiener-Hopf operator, 131, 256

 X^+ 1-point compactification of locally compact X , 266
 $\overset{\circ}{X}$ interior of a manifold with boundary, 184
 $[X, Y]$ homotopy set, 72

 $\text{YM}(\omega)$ Yang-Mills functional, 462

 $Z^2(\mathcal{U}; \mathbb{Z}_2)$ group of Čech 2-cocycles with values in \mathbb{Z}_2 relative to the cover \mathcal{U} , 527
 \mathbb{Z}^n n -dimensional lattice, 140
 (\cdot, \cdot) L^2 -inner product on $\Omega^k(P \times_G W)$, 404
 $(\cdot, \cdot)_0$ inner product in $L^2(X; E)$, 170
 $\langle \cdot, \cdot \rangle$ inner product in Hilbert space, 3

- $\langle \cdot, \cdot \rangle_h$ Hermitian metric on vector bundle, 170
- $[\cdot, \cdot]$ bracket (commutator), 515
- $[\dots]$ = span(\dots) linear span of vectors, 21, 400
- * star
 - convolution, 706, 708, 711
 - Hodge star operator on forms, 172, 405
 - star operator on exterior algebra, 172
 - taking the adjoint operator, 15
- \boxtimes outer tensor product, 243, 261, 267, 268, 278, 279
- \cup_f gluing
 - two manifolds by a diffeomorphism of their boundaries, 191
 - two vector bundles by an isomorphism over the intersection of their bases, 718
- $|\alpha|$ degree of multiindex α , 135
- $\|\cdot\|$ norm
 - L^2 -norm on $C^\infty(P \times_G W)$, 405
 - $|\cdot|_e$ norm relative ds^2 , 474
 - $|\cdot|_h$ norm relative $h = f^2 ds^2$, 474
 - $\|\cdot\|_1$ trace norm on \mathcal{I}_1 , 60
 - $\|\cdot\|_k$ Sobolev norm, 37
 - $\|\cdot\|_{k,K}$ semi norm on $C^\infty(\mathbb{R}^n)$, 136
 - $\|\cdot\|_{p,k}$ L^p Sobolev norm, 497
 - operator norm, 3
 - Sobolev norm, 195
- $\langle \cdot, \cdot \rangle$ $C^\infty(M)$ -valued pairing on $C^\infty(P \times_G W)$, 404
- $\otimes A, \cdot$ B pairing
 - $H^q(X; \mathbb{Z}) \times H_q(X; \mathbb{Z}) \rightarrow \mathbb{Z}$, 644
- \wedge Fourier transform
 - Fourier integral, 26, 708
 - Fourier series, 706
- \circ dotted
 - for vector bundles, 182
 - interior of manifold with boundary, 184
- ∇ nabla operator
 - ∇^E connection (covariant differentiation operator) on vector bundle E , 177
 - ∇^2 invariant second derivative, 536
 - $\nabla^{\omega \oplus \theta}$ covariant derivative on $T^{r,s}(W)$, 435
 - ∇u gradient of function u , 145
- \oplus direct sum, 5
- \otimes tensor products, 166
- \perp orthogonal, 13
- \pitchfork transversality, 700
- \sim asymptotic expansion, 549
- \sim homotopy of maps, homotopy equivalence, 72, 716
- \smile cup product, 311
- \times Clifford multiplication
 - see also \mathbf{c} , 328
- $(\wedge)^\vee$ Bokobza-Haggiag inverse Fourier transform, 233
- $\vee: S^2(\mathbb{R}^n) \times S^2(\mathbb{R}^n) \rightarrow \mathcal{R}(\mathbb{R}^n)$ vee (Kulkarni-Nomizu) product, 428
- \wedge exterior multiplication (wedge product), 171
- \wedge Bokobza-Haggiag (global) Fourier transform of section u on Riemannian X in Hermitian bundle with fixed connection and fixed bump function, 232
- $\cdot \lrcorner$ (left) interior multiplication, 173, 519
- $\cdot \lrcorner$ (left) exterior multiplication, 173, 519

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Praise for *Index Theory with Applications to Mathematics and Physics*:

This book ... describes in impressive detail one of the greatest achievements of 20th-century mathematics, the Atiyah-Singer index theorem. As a bonus, the author includes an extensive and detailed discussion of some of the applications of the theorem and more recent developments in the direction of gauge theoretic physical models. For the student, the gold lining in the book is that all the background is there—starting at a level that is accessible to anyone with a reasonable mathematical education. There is also a bridge to the point of view from the physics side. The student can dip into the book at many points and use it to learn individual topics. The discussion covers a very broad sweep of mathematics which will be of interest to anyone aiming to understand geometric analysis, whether or not gauge theories are their primary focus. Two proofs of the Atiyah-Singer theorem are given, one based on K-theory and the other on the heat kernel approach... The authors have included many discussion sections that illuminate the thinking behind the more general theory... There is much valuable historical context recorded here. Moreover, it captures the flavour of contemporary mathematics, with the interaction of geometric, topological, analytic, and physical perspectives and methods being in the forefront of the discussion.

Alan Carey

Mathematical Sciences Institute at Australian National University, Canberra

Those wishing to start their study of the theory of elliptic operators will appreciate the book's use of rigorous exposition intermingled with intuition-building casual and historical discussion. Experts will find many features of the book useful and convenient.

Paul Kirk

Indiana University Bloomington

The real strength of the book is its detailedness, which makes it particularly attractive for the learning (graduate) student. But the senior researcher, likewise, will treasure this somewhat unconventional textbook as a very valuable source of information.

Matthias Lesch

Universität Bonn, Germany

Two or three famous Index Formulas discovered and proved in the course of middle decades of the last century are some of the highest peaks in a mountain country rising from the vast plains of functional analysis, theory of smooth manifolds, and homotopical topology.

This treatise, written with ambition, wit and (mathematical) eloquence, strives to combine the qualities of a guide-book, historical chronicles, and a hiking manual for enthusiastic travellers and budding future explorers of this vast territory.

Any reader possessing will and enthusiasm can profit from studying (parts of) this book and enjoy finding his or her own path through this land.

Yuri I. Manin

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My struggles with this theorem are vivid in my mind. I still remember the sunny autumn afternoon when I picked from the library the first edition of this book. From the first moment I opened it I thought it spoke to me, the beginning graduate student with a limited mathematical experience. The proofs had the level of detail I needed at that stage in my life. What awed me most was the wealth of varied examples from exotic worlds I did not even suspect existed. Reading those examples and the carefully crafted proofs I began to get a glimpse at the wonderful edifice behind the index theorem.

The present edition has the same effect on the more mature me. And it does quite a bit more. Old examples are refined, and new ones added. Facts whose proofs were only sketched in the old edition are now given complete, or almost complete proofs. The chapter on gauge theory is completely and massively rewritten and it incorporates some of the spectacular developments in this area that took place in the intervening time. As importantly, throughout the book, the exposition is sprinkled with many mathematical "anecdotes" which give the reader a glimpse into the minds of

the pioneers of the subject.

I believe that any youngster who will pick this new edition will be as grateful as I am to the authors for the care and their concern for the reader. The teacher of this subject, as well, will have many things for which to be grateful: this is one of the few places containing such a wealth of examples and care for detail.

Liviu Nicolaescu
University of Notre Dame, Indiana

The present book is a landmark, being a complete introduction to practically all aspects of the Index Theorem, complete with basic examples and exercises, covering topological ideas: homotopy invariance and K-theory; analysis: elliptic operators and heat equations; geometry: principal bundles and curvature; physics: gauge theory and Seiberg-Witten theory. The first English version appeared in 1985, but the present book is a largely reworked and much expanded treatise; it is a highly informative presentation, based on the authors' expert knowledge and pedagogical interests. Indeed, it seems to be presently the most complete single book on the many approaches to and applications of the Atiyah-Singer Index Theorem. Experts as well will find inspiration in this book.

Bent Ørsted
Aarhus University, Denmark

The student who wants to explore the whole shape of this huge and complex territory—as well as the usual climbing routes to the summit of the Index Theorem itself—can hardly do better than to enlist the genial and enthusiastic guide service of Bleecker and Booss-Bavnbek.

John Roe
Pennsylvania State University

Professors Bleecker and Boosß-Bavnbek have followed ... developments in index theory from the beginning, and made original contributions of their own... Assuming only basic analysis and algebra, [this book] gives detailed constructions and proofs for all the necessary concepts, along with illuminating digressions on the various paths through the rich territory of index theory.

Robert Seeley
Professor Emeritus, University of Massachusetts, Boston

The very appreciable feature of the book is its discussion of the applications of the theorem in low-dimensional topology and in gauge theories. For these reasons, the book will be a valuable reference for theoretical and mathematical physicists, especially those interested in the foundations of quantum gauge theories, at the basis of the standard model of elementary particle physics. Mathematically oriented graduate students will greatly profit from reading this excellent book.

Franco Strocchi
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Students of mathematics and physics will find this book to be an excellent resource for study of this vast subject. In particular, attention is given to exposition of the subject from different angles, which is very helpful as a bridge between physics and mathematics.

Cumrun Vafa
Donner Professor of Science, Physics Department, Harvard University

Readers from a wide range of backgrounds will find much to learn here.

Edward Witten
Institute for Advanced Studies, Princeton, New Jersey
