

# Hang Lung Mathematics Awards



數學科學研究所  
The Institute of Mathematical Sciences



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The Chinese University of Hong Kong



恒隆地產  
HANG LUNG PROPERTIES



# Hang Lung Mathematics Awards *Collection of Winning Papers 2008*

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Hang Lung Mathematics Awards  
Collection of Winning Papers, 2008

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## Preface

Since the inaugural Hang Lung Mathematics Awards (HLMA) held in 2004, we have been extremely pleased by the enthusiasm and interest displayed among students, teachers and secondary schools who have participated. 2008 HLMA marked the third occasion of the competition held in Hong Kong.

Before the launch of the competition in 2004, we were concerned whether students and teachers in Hong Kong would be interested in this type of research-based competition, and whether it would be too demanding of the students. In hindsight, we are pleased to say that those concerns have been unfounded. Students and teachers have capably risen to the challenge, demonstrated creativity in selecting their research topics and displayed remarkable ingenuity in applying mathematics to solve problems or prove a proposition.

This book has gathered the research papers of the winners of the 2008 HLMA which have reached a high level of academic standards in terms of methodology, execution and scholarship. It is exciting to witness students' tackling a wide variety of challenging mathematical problems with great skill and imagination. They seemed to have discovered how enjoyable mathematics can be.

We are also proud to see that HLMA has been winning tremendous and continuous support from not only the local, but also the international academic community since its establishment. It is our hope that HLMA will continue to motivate youngsters to think more critically and induce their interest in mathematics. We also yearn to position HLMA as a pioneering model encouraging the business sector to support other academic pursuits, so that the government, the business sector and the education community can join forces to discover talents for our society.

Professor Shing-Tung Yau

Director, The Institute of Mathematical Sciences  
The Chinese University of Hong Kong

Mr. Ronnie C. Chan

Chairman  
Hang Lung Properties Ltd.

October 2012

## Acknowledgement

The publishing of this book has been made possible by donations from Hang Lung Properties Ltd.

The contribution to mathematics education from the initiators of Hang Lung Mathematics Awards, Mr. Ronnie C. Chan and Professor Shing-Tung Yau, will be marked in history. Professor Yau provides the academic leadership and vision for the Awards and he is also one of the editors of this volume. Hang Lung Properties Ltd. has been providing the financial support for the Awards and additional funding for the editorial and publishing cost of this volume.

The reputation of an academic award is always established upon its rigorous process of assessment and review. The Scientific Committee is the cornerstone of this intellectual excellence. The Screening Panel would like to thank Wenyi Chen, Feng Hui, Jiaxin Hu, Jianguo Huang, Wen Huang, Qinghui Liu, Jihua Ma, Kung-Fu Ng, Yanhui Qu, Hui Rao, Song Shao, Yuguang Shi, Baorui Song, Bo Tan, Zhenhan Tu, Gengsheng Wang, Jiayan Yao, Changjian Zhang, Guobiao Zhou, Xiaofeng Zou for their professional judgment.

The Steering Committee has contributed greatly to the popularity of the Awards in society. Many students, teachers, and schools have participated in the competition. The Executive Committee, chaired by Thomas Au, is indebted to the secretariat led by Serena Yip and the assistance of Peony Wu and Simon Tse. The cooperation of Susan Wong, Joyce Kwong, Joyce Leung, and the newly on board Carolina Yip from Hang Lung Properties has been invaluable.

This volume will never be perfect without the commitment of Kung-Fu Ng who have led the editorial work. He would like to express his sincere gratitude to Chi-Ping Lau and Ping-Kwan Tam for their careful reading and comment on the winning papers. The technical work of Yat-Ming Cheung and Chun-Ngai Cheung and the coordination of Mavis Chan had also played a crucial role in the editorial process.



# Hang Lung Mathematics Awards

**Introduction** The Hang Lung Mathematics Awards (HLMA), co-organized by Hang Lung Properties Ltd. and the Department of Mathematics at The Chinese University of Hong Kong, is a biennial research-based mathematics competition for secondary school students in Hong Kong. Founded in 2004 by Mr. Ronnie C. Chan, Chairman of Hang Lung Properties Ltd., and world-renowned mathematician Professor Shing-Tung Yau, a 1982 Fields Medalist and 2010 Wolf Prize recipient, the competition aims to stimulate creativity and to encourage intellectual discovery in mathematics and science among secondary school students in Hong Kong.

Schools are invited to form teams and, under the supervision of a lead teacher, the teams design and carry out a mathematics research project. Each team submits a project report summarizing the findings, which is evaluated by the Scientific Committee in a multi-step process similar to that for the selection of publication in a scientific journal. Short-listed teams are invited to participate in an oral defense of their project before members of the Scientific Committee. This final stage is modeled after a doctoral degree defense and comprises two parts: a public presentation of the research project followed by a closed-door inquiry. The winners of the HLMA will be decided after the oral defense.

Hang Lung Properties Ltd. donated over HK\$2 million to each competition. The Department of Mathematics at The Chinese University of Hong Kong provides tuition scholarships to the teachers of the winning schools. The Department of Mathematics handles all administrative, operational, and educational aspects of the competition.

**Participation** Up to five students of the same secondary school may form a team to participate in the competition. The team shall be led by a teacher of the school. After a simple registration process which started in early 2007, each team performed their study and research on a topic selected by the team. Research reports were submitted in August 2008. There were 77 teams of 229 students from 50 schools participated and 50 of those teams submitted research reports.

**Assessment** To decide the winners, there are two stages of assessment: research report review and oral defense. The assessment in each stage is the jurisdiction of the Scientific Committee, with the support of the Screening Panel and external experts from the international mathematics community. The review spanned from early August to early October of 2008. In this stage, each research report has to pass an initial screening. Then it is sent to at least two referees of external experts. A shortlist will be selected by the Scientific Committee to proceed to the second stage of assessment, which is the oral defense. In this stage, each team will make a brief presentation of their research in front of the Scientific Committee. The presentation is open to the public. It is then followed by a closed door inquiry. The winners will be decided afterwards.

**Awards** Up to eight awards are presented to the mathematics projects that meet the highest academic standard in terms of research methodology and originality: Gold, Silver, Bronze, and up to five Honorable Mentions. Each award consists of four components: a “Student Education Award” to be shared equally among team members and applied towards their university studies; a “Teacher Leadership Award” for the supervising teacher; a “School Development Award” to promote mathematics education at the school; and a “Tuition Scholarship” for any teacher at the winning schools to earn a Master of Science (MSc) in Mathematics from The Chinese University of Hong Kong. The winning students and teachers also received a crystal trophy and a certificate while the school was presented with a crystal trophy.

## Organization

The two principal committees of the Hang Lung Mathematics Awards are the Scientific Committee and the Steering Committee.

### Scientific Committee, 2008

The Scientific Committee comprises world renowned mathematicians and is the academic and adjudicating body of the Hang Lung Mathematics Awards. The Scientific Committee oversees the whole process of assessment, including the review of the research project reports and conducting the oral defense. The committee upholds the academic integrity and high standard of the mathematics research competition as well as the awards.

**List of members** (affiliations at the time of the event)

*Chair:* Professor Shing-Tung Yau, Harvard University  
Professor Tony F. Chan, University of California, Los Angeles  
Professor David C. Chang, Polytechnic Institute of New York University  
Professor Chong-Qing Cheng, Nanjing University  
Professor John H. Coates, University of Cambridge  
Professor Benedict H. Gross, Harvard University  
Professor Ka-Sing Lau, The Chinese University of Hong Kong  
Professor Jian-Shu Li, The Hong Kong University of Science and Technology  
Professor Chang-Shou Lin, National University of Taiwan  
Professor Jill P. Mesirov, Broad Institute of MIT and Harvard  
Professor Kenneth C. Millett, University of California, Santa Barbara  
Professor Ngaiming Mok, The University of Hong Kong  
Professor Stanley J. Osher, University of California, Los Angeles  
Professor Duong H. Phong, Columbia University  
Professor Wilfried Schmid, Harvard University  
Professor Tom Yau-Heng Wan, The Chinese University of Hong Kong  
Professor Hung-Hsi Wu, University of California, Berkeley

**Screening Panel** is a subcommittee of the Scientific Committee. It handles the initial screening of each report, supervises external review process, and serves as a bridge between all referees and members of Scientific Committee.

*Chair:* Professor Tom Yau-Heng Wan, The Chinese University of Hong Kong  
Professor Wing Sum Cheung, The University of Hong Kong  
Professor Conan Nai Chung Leung, The Chinese University of Hong Kong

## Steering Committee, 2008

The Steering Committee comprises mathematicians and representatives from different sectors of society and serves as the advisory body. The committee also includes mathematics department heads of major Hong Kong universities. Some members from the Scientific Committee and Executive Committee also serve in the Steering Committee so that it has an overall perspective of all the aspects.

### **List of members** (affiliations at the time of the event)

*Chair:* Professor Sir James A. Mirrlees, 1996 Nobel Laureate in Economics  
Professor Thomas Kwok-Keung Au, The Chinese University of Hong Kong  
Professor Wing-Sum Cheung, The University of Hong Kong  
Professor Ka-Sing Lau, Chairman, Mathematics Department, CUHK  
Professor Jian-Shu Li, The Hong Kong University of Science and Technology  
Professor Lo Yang, Chinese Academy of Sciences  
Mr. Siu-Leung Ma, CEO, Fung Kai Public Schools  
Mr. Chun-Kau Poon, Principal, The Hong Kong Federation of Youth Group  
Lee Shau Kee College  
Ms. Susan Wong, Hang Lung Properties Limited  
Mr. Chee-Tim Yip, Principal, Pui Ching Middle School

**Executive Committee** is a subcommittee of the Steering Committee. It is in charge of the administration, promotion, team registration, communication, and organization of events including the oral defense.

*Chair:* Professor Thomas Kwok-Keung Au, The Chinese University of Hong Kong  
Dr. Ka-Luen Cheung, The Hong Kong Institute of Education  
Dr. Leung-Fu Cheung, The Chinese University of Hong Kong  
Dr. Charles Chun-Che Li, The Chinese University of Hong Kong  
*Secretariat:* Ms. Serena Wing-Hang Yip, The Chinese University of Hong Kong

## **Gold, Silver, and Bronze**

### **GOLD**

Team member(s): Kwok Chung Li, Chi Fai Ng  
School: Shatin Tsung Tsin Secondary School  
Team Teacher: Mr. Wing Kay Chang  
Topic: *Isoareal and Isoperimetric Deformation of Curves*

### **SILVER**

Team member(s): Chi Yeung Lam, Yin Tat Lee  
School: The Methodist Church Hong Kong Wesley College  
Team Teacher: Mr. Chun Kit Ho  
Topic: *Sufficient Condition of Weight-Balance Tree*

### **BRONZE**

Team member(s): Fung Ming Ng, Chi Chung Wan, Wai Kwun Kung,  
Ka Chun Hong  
School: S.K.H. Tsang Shiu Tim Secondary School  
Team Teacher: Mr. Yiu Kwong Lau  
Topic: *Fermat Point Extension – Locus, Location and Local Use*

