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Advanced Lectures in Mathematics
Volume XXIII

Recent Developments in Geometry and Analysis

edited by

Yuxin Dong · Jixiang Fu · Guozhen Lu

Weimin Sheng · Xiaohua Zhu

 International Press
www.intlpress.com

 高等教育出版社
HIGHER EDUCATION PRESS

Advanced Lectures in Mathematics, Volume XXIII
Recent Developments in Geometry and Analysis

Volume Editors:

Yuxin Dong (Fudan University)
Jixiang Fu (Fudan University)
Guozhen Lu (Wayne State University)
Weimin Sheng (Zhejiang University)
Xiaohua Zhu (Peking University)

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ISBN: 978-1-57146-250-3

Printed in the United States of America.

16 15 14 13 12 1 2 3 4 5 6 7 8 9

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Dedicated to
Professors Zheng-guo Bai and Yibing Shen

Preface

This is a special volume dedicated to Professor Zhengguo Bai (aka Chenkuo Pa) on the occasion of his 95th birthday and to Professor Yibing Shen on the occasion of his 70th birthday. As friends and students of Professors Bai and Shen, it is our great pleasure to edit this volume to celebrate their mathematical lives and careers.

In addition to their own important mathematical contributions in projective geometry, Riemannian and differential geometry and Finsler geometry, Professors Bai and Shen have made tremendous contributions in educating and training of geometers in China. Professors Bai and Shen have trained around 80 M.S. and Ph.D. students over the years, including over 60 students jointly supervised by them. We refer the reader to “A list of graduate students of Professors Bai and Shen”. We also refer the reader to “A brief introduction of Professor Zhengguo Bai” and “A brief introduction of Professor Yibing Shen” for more information about their mathematical lives.

This volume describes some recent development in differential geometry and geometric analysis. Papers in this volume address a wide array of research topics. We are confident that this volume will be useful for geometers and analysts and mathematicians working in related areas. In particular, it can be very beneficial to graduate students, postdoctors, and young mathematicians who are just beginning their research in these areas.

We would like to take this opportunity to thank wholeheartedly contributors of papers in this volume. This volume cannot be in the present form without their hard work and mathematical contributions. We like to thank C. Sogge, Editor-in-Chief of American Journal of Mathematics, as well for his short tribute to Professor Zhengguo Bai who published four papers in the American Journal of Mathematics in the 1940's. We also thank Editors of the series of Advanced Lectures in the Mathematics for their support in the process of editing of this volume, in particular to Lizhen Ji for his initial invitation and solicitation to have this volume in honor of Professors Bai and Shen, and his constant encouragement and valuable suggestions. Special thanks are also due to Ms. Liping Wang and Ms. Huaying Li from the Higher Education Press for their tireless support and hard work. Without the devotion of Ms. Wang and Ms. Li, we could not have had completed this well organized volume in due time.

Lastly, it is our great honor to be in charge of editing this volume in celebration of the mathematical lives of Professors Bai and Shen. As former students and friends of Professors Bai and Shen, we would like to take this opportunity to express our own sincere appreciation to them for their constant teaching, support to our mathematical careers, their long-lasting friendship. We truly admire their mathematical contributions and the enormous impact of their training of differ-

ential geometers, who have become by now a major composition of researchers in these ares in China.

Yuxin Dong, Fudan University, China
Jixiang Fu, Fudan University, China
Guozhen Lu, Wayne State University, USA
Weimin Sheng, Zhejiang University, China
Xiaohua Zhu, Peking University, China

May, 2012

A Tribute by Christopher Sogge to Professor Zhengguo Bai

Professor Zhengguo Bai grew up in difficult times from a humble background and was largely self-taught. Unlike most other Chinese mathematicians of his time he did not study abroad. He did not receive a Ph.D., but, on the basis of his research, was promoted to Professor at Zhejiang University and later became the Department Chair of Hangzhou University. The war with Japan greatly affected Zhejiang University and caused it to be moved from Hangzhou. Afterwards, Professor Bai was one of the top leaders responsible for the revival of Hangzhou as a top center of mathematics. He also was an excellent and dedicated teacher who trained dozens of important Chinese geometers.

Professor Zhengguo Bai also published papers at the beginning of his career under the name of Chenkuo Pa. These included many in top mathematical journals. Among these are several that were published in the American Journal of Mathematics in the 1940's. The latter include an important series of papers in the beginning of that decade that solved a famous problem Fubini, which followed earlier work of Blaschke, in projective differential geometry and ruled surfaces. He also studied projective minimal surfaces of coincidence, including quadrics of Boutard, Gordeaux sequences of quadrics, which resulted in other papers published in the American Journal of Mathematics. The papers speak for themselves, but, at that time, it was rare for Chinese mathematicians to be publishing in international journals of that caliber.

As the Editor-in-Chief of the American Journal of Mathematics and someone with close ties to Zhejiang University, it is an honor to write this short tribute to Professor Bai who has contributed so much both through his research articles which were published in our journal and many other top journals and through his scholarship and teaching.

Christopher Sogge¹
Baltimore, Maryland, USA
May 27, 2011

¹Christopher Sogge is J. Sylvester Professor at the John Hopkins University. He was an invited speaker of the ICM in 1994, a Presidential Young Investigator Award recipient in 1989 and received a Guggenheim Fellowship in 2005. He is currently an overseas distinguished professor of Zhejiang University awarded by the Ministry of Educations of China. He has been the Editor-in-Chief of the American Journal of Mathematics since 2005.

A Brief Introduction of Professor Zhengguo Bai (aka Chenkuo Pa)

Professor Bai was born on December 17, 1916. His father managed a small pharmacy in the town to support the family. His father died when Bai was 3. As a result, his mother had to move with him to a village. Bai attended the elementary school in the village when he was 8. The whole school had only one teacher and Bai was mainly taught Chinese and literature. When Bai was in the third grade, the school hired a mathematics teacher and Bai started to learn mathematics. Bai was highly praised by his mathematics teacher for his mathematical talent. In 1933, Bai graduated from a high school. He was prepared to quit further school training because of his family financial difficulty and found a job as an elementary school teacher. Right before he started his job, he was informed that he did extraordinarily well in the Zhejiang Province Middle School Graduation Examination and was ranked number one in the whole district. His Pingyang county fellows were happy to voluntarily support him financially for him to continue his study at Wenzhou High School. Bai worked very hard during his high school study. He was most interested in mathematics. During his three year study, he studied Calculus and Projective Geometry. In particular, he was very interested in the books *Geometric Proof Method* of Jichi Yan and *Conic Sections* of Salmon. His mathematical teacher who studied abroad in Japan paid special attention to Bai. He told Bai, “there were two famous mathematicians Lifu Jiang and Buchin Su from Wenzhou. Professors Buchin Su and Jiangong Chen studied in Japan and are very famous in Japan. These two professors are now teaching at Zhejiang University. Since you are so talented in mathematics, you should apply for admission to Zhejiang University after graduation” His mathematics teacher inspired him to study further in mathematics and played an important role in his future career orientation.

In 1936, Bai was admitted to the department of mathematics at Zhejiang University. In a question concerning finding a parabola passing through four points, he solved two parabolas. This got the attention of Professor Buchin Su. At the time, Dr. Kezhen Zhu was the president of Zhejiang University and Professors Su and Chen had very high reputation at the university. Professors Su and Chen strived to teach all the important mathematics courses by themselves. This has laid down a solid mathematical foundation for Bai and trained his scientific research ability. In 1937, the anti-Japanese war started. Zhejiang University began to move to the West of China and finally to Zunyi in Guizhou Province. The college of sciences was moved to Mitang, Guizhou in 1941. Bai graduated from Zhejiang University in 1940 in Zunyi, Guizhou and was hired as a teaching assistant. He chose to do research in the area of projective geometry. Bai’s advisor Professor Buchin Su was

already very famous in this research area at the time. A key player in this area was G. Fubini, an Italian mathematician. Most papers of Fubini and other mathematicians in this area were written in Italian and French. Bai started to learn Italian and French. From 1941, Bai published a series of papers in this direction. These papers were completed during the very difficult time of the anti-Japanese war.

Bai's first paper was published in *Science Record* by the Chinese Mathematical Society, the only mathematical research journal in China then. The journal adapted a high standard on publishing papers in English. Due to then the difficult time of inflation, the journal was printed in poor quality coarse papers. Bai still has a copy of a reprint of that paper. Because of Bai's outstanding research achievement, he was hired as a research assistant at Zhejiang University in 1943. At the time, Peking University, Tsinghua University and Nankai University, the then Southwest Union University, moved to Kunming in Yunnan Province and established the position of Research Assistant. President Kezhen Zhu of Zhejiang University hired Bai as a Research Assistant so that Bai could focus on doing research. That was the only case at Zhejiang University then. In 1945, Bai learned from *Mathematical Review* that his papers were published in the *Transactions of American Mathematical Society*, and *American Journal of Mathematics*, etc. Zhejiang University then recommended Bai to the Ministry of Education to promote Bai to the position of Instructor. At the end of the anti-Japanese war, Bai's published papers in US were mailed back to him. These research papers earned him a very high reputation at Zhejiang University. In 1948, Bai was promoted to the rank of Associate Professor. This was a very rare case within Zhejiang University for someone who was just eight years after graduation and who had never studied abroad to be promoted to this position.

Bai became eminent for his solution to the problem Fubini on projective differential geometry in the early 1940's. In the 1930-40's, there were three universally acknowledged schools of projective differential geometry: the American school, the Italian school and the Zhejiang University school led by Buchin Su. Bai was one of the representatives of Su's school. At that time, an open problem of projective differential geometry had stimulated a substantial interest among the mathematicians: Does there exist a surface such that a family of its asymptotic curves are projectively equivalent? This problem comes from a theorem of the well known German mathematician W. Blaschke, which says that, if all asymptotic curves of one system on a non-ruled surface belong to a linear complex, then they are projective to each other.

G. Fubini, the renowned Italian Mathematician then studied the converse problem of the above Blaschke theorem and showed the validity for a ruled surface. Thus Fubini raised the well known problem Fubini: With the exception of the surfaces a family of whose asymptotic curves belong to a linear complex, whether does there exist a non-ruled surface such that a family of its asymptotic curves

are projectively equivalent. Bai had overcome all the roadblocks in the solving of the problem Fubini and gotten an affirmative answer. He showed that such a non-ruled surface is a surface whose one family of asymptotic curves belong to a linear complex or of coincidence a special class of (i.e., ∞^2) projective minimal surfaces. This accomplishment was recognized by Fubini himself who requested the press to publish in advance Bai's paper. Later on, Bai's this result was recorded in the book *Obituary: Guido Fubini. 1879—1943* by A. Terracini and was also introduced in a specific chapter in Su's monograph *The General Projective Theory of Surfaces*.

In addition to this important work, Bai contributed many works of originality in projective difference geometry, such as on the Moutard quadrics, on the Godeaux sequence of quadrics, and on the projective theory of surfaces in a ruled space. He published about ten significant papers in this direction, including four in the American Journal of Mathematics, one in the Transactions of American Mathematical Society, one in Bulletin of the American Mathematical Society.

Starting from 1950's, Bai started to study the differential geometry of generalized spaces. In 1957, he published a paper *on the integral curvature of a curvilinear polygon* in \mathbb{R}^n which generalized the well known F. Fenchel's theorem. The result is as follows. Let C be a curvilinear polygon in \mathbb{R}^n with the interior angles $\theta_1, \theta_2, \dots, \theta_l$. Then its total curvature κ satisfies the inequality

$$\int_C \kappa ds + \sum_{i=1}^l \theta_i \geq 2\pi.$$

The equality holds only for a plane convex curvilinear polygon. This significant result about space curves was recorded in to the book *Ten-Year Chinese Mathematical Achievements* as well as in the Math Chapter of *China Encyclopedia*.

In 1964, Bai completely determined the first fundamental form of a Riemannian manifold which admits several independently infinitesimal concircular transformations. This is a fundamental problem in concircular geometry raised by the well known Japanese Geometer K.Yano. M. Berger, the famous French geometer requested from Bai an offprint of this result in 1980, when the international conference on differential geometry and differential equations was held in Beijing.

Bai also studied independently the Codazzi-Ricci equation and the Gauss equation of a submanifold in a Riemannian manifold, characteristics of the curvature tensor of a conformally flat Riemannian manifold with constant curvature, and the hypersurfaces with constant curvature of a conformally flat Riemannian manifold. He got several important results written in more than ten papers which were published in China and abroad.

Bai's research activities were interrupted during the era of "Cultural Revolution". After the "Cultural Revolution", Bai began interested in the global Riemannian geometry and studied systematically on Riemannian manifolds with quasi-constant curvature. For example, he proved that a Riemannian manifold

admits an isometric imbedding into two Riemannian manifolds, respectively, with distinct constant curvatures if and only if it is with quasi-constant curvature. This is a very interesting and unprecedented result. The similar result was later on obtained by M. do Carmo, a famous Brazilian geometer. On the other hand, Bai also got several very nice results on global submanifold geometry.

In 1952, after the reassembly of the national universities, Bai started to work at the newly established Zhejiang Normal College. The research and teaching environment at the Department of Mathematics was very poor. Not only were there insufficient faculty members, but also very few library books or journals. It was virtually impossible for faculty members to do any quality research. It was even difficult to offer advanced courses for undergraduates in mathematics. Bai was the head of the Geometry Section of the department. He organized seminars for faculty members. A few years later, the overall quality of faculty members has improved. This laid down a solid foundation for the founding of the Hangzhou University Mathematics Department a few years later. In 1956, Professor Buchin Su invited Prof. Bai to work at Fudan University. In 1958, Hangzhou University was founded and Professor Jiangong Chen was appointed as the Vice President and Prof. Bai was appointed as the Chairman of Mathematics Department. In the same year, Hangzhou University and Zhejiang Normal College were merged into the new Hangzhou University. Bai also started to study Riemannian Geometry. In 1962, Prof. Bai was approved by the State Council of China to be eligible to advise three-year term graduate students. He and Professor Chen each recruited five graduate students. These graduates of Prof. Bai have later become successful mathematicians. For instance, Professor Yibing Shen of Zhejiang University, Professor Chongzhen Ouyang (former chair of mathematics department at Nanchang University), Professor Kairen Cai (former Dean of Library of Hangzhou Normal University).

During the “Cultural Revolution”, Prof. Bai has to stop advising graduates and sent to do research on the numerical forecasting of Tornado at the Zhejiang Province Meteorological Observatory. The research group led by Prof. Bai completed a research paper which was awarded the Second Prize of Zhejiang Provincial Department of Science and Technology. After the ending of “Cultural Revolution”, Prof. Bai was once again appointed as the Chair of the Mathematics Department of Hangzhou University. He became the honorary chairman of the department starting 1982.

In 1978, Professor Bai was approved by the State Council of China to be eligible to supervise Master of Sciences students in mathematics. In 1981, Professor Bai was approved by the State Council of China to be eligible to supervise doctoral students in mathematics. Professor Bai was among the first group of mathematicians who were granted this honor in China. In 1991, Professor Bai was approved by the State of Council of China to receive special allowance and he was also approved by the Personnel Department of China to be a permanent professor

(not subject to retirement). Professor Bai is currently still a tenured professor at Zhejiang University.

Professor Bai made tremendous contributions in training geometers in China. Right after the anti-Japanese war, Professor Buchin Su was appointed the president of Taiwan University in Taiwan, China. Professor Bai took over the teaching duties of two major geometry courses “Differential Geometry” and “Coordinate Geometry” at then Zhejiang University till 1952. During those years, Students who took Professor Bai’s classes include famous mathematicians and members of Chinese Academy of Sciences Professors Yuan Wang and Chaohao Gu, topologist Professor Zhongdao Yang, and Professors Guangchang Dong and Zhurui Guo.

After “Cultural Revolution”, Professor Bai started to supervise Master of Sciences and Doctoral Students in differential geometry jointly with Professor Yibing Shen. Together, they have trained many outstanding geometers. From 1982, Shen and professor Bai supervised many M.S. graduate and Ph.D. students. These students, just to mention a few, include the following M.S. students Zhenqi Li, Guofang Wang, Qiyuan Chen, Xiaoxiang Jiao, Ruixia Song etc. and Ph.D. students Yuxin Dong, Xiaohuan Mo, Xiaohua Zhu, Weimin Sheng, Jixiang Fu, Xi Zhang, Qun He, etc. They all accomplished very well. In addition, Guozhen Lu who went to study at Rutgers University in USA soon after being admitted to become a M.S. student of Professors Bai and Shen, and Senchun Lin and Meiling Gao who went to study at Rutgers University in USA after receiving their M.S. degree all did very well in the USA now. Gao Meiling and Lin Senchun are now Senior Software Engineers at Oracle Corporation and Metlife in USA respectively. Among those students of Professors Bai and Shen who are working in the academia, Yuxin Dong (the first doctoral student of Professors Bai and Shen) and Jixiang Fu are professors at Fudan University; Xiaohuan Mo and Xiaohua Zhu are professors at Peking University; Weimin Sheng is a professor at Zhejiang University; Xi Zhang is a professor at the University of Science and Technology of China; Qun He is a professor and Associate Chair of Mathematics Department at Tongji University; Guofang Wang is a professor at Otto-von-Guericke-Universitat Magdeburg in Germany; Xiaoxiang Jiao is a professor at the Graduate School of Chinese Academy of Sciences; Zhenqi Li was a recipient at Fudan University of the 100 national outstanding doctoral dissertations and now is a professor and former Chairman of Mathematics Department at Nanchang University. Xiaohua Zhu and Jixiang Fu were both awarded the National Outstanding Young Scholars of China. Xiaohua Zhu is also a Changjiang Distinguished Professor of Ministry of Education of China. He also received the Qiushi Outstanding Young Scholar Award from Hong Kong Qiushi Foundation, the First Prize for the ICTP outstanding young scholar award in Science and Technology in Italy, and the First Prize of Natural Science Award of the Ministry of Education of China. Jixiang Fu is also a winner of Silver Medal of Morning Science mathematics award at ICCM and an invited speaker at the 2010 ICM. Guozhen Lu is now a professor at Wayne

State University in USA and a Distinguished Professor at Beijing Normal University. Lu is also a Commissioner of Michigan Asian Pacific American Affairs Commission in USA, and currently serving as the Board Chairman of Zhejiang University Education Foundation (USA). All of these students of Professors Bai and Shen have made valuable contributions to the areas of Differential Geometry and Geometric Analysis.

Professor Bai held a number of positions in his career. They include the positions as Chairman and Honorary Chairman of the Mathematics Department at Hangzhou University, Vice Chairman of the Scientific Committee of Hangzhou University, President and Honorary President of the Zhejiang Province Mathematical Society, among many others. He also served on the editorial boards of Chinese Annals of Mathematics, Mathematical Research and Review, National University Textbook Science Section, Chinese Encyclopedia Mathematics Section, etc. He was a recipient of numerous awards including the First Prize of the Outstanding Research Papers Award of Zhejiang Provincial Science and Technology Committee in 1986, the First Prize of Natural Science by Zhejiang Provincial Education Committee, the Third Prize of Progress in Science and Technology of Ministry of Education of China in 1990, and the Second Prize of National Universities Excellent Textbooks Award, etc.

Yibing Shen at Zhejiang University

A Brief Introduction of Professor Yibing Shen

On October 21, 1939, Yibing Shen was born in a business family in Jiaxing city, Zhejiang province. In the age full of turmoil, his family lived in a very hard life. As he was the only child in the family, he had doting parents, but since his parents attached importance to child's education, his education during the childhood was not affected too much. In September, 1952, Yibing Shen enrolled in No. 2 Middle School of Jiaxing, which had been Xiuzhou Middle School of Jiaxing, a missionary school founded by American missionaries. And it was where a mathematics master, Shiingshen Chern, spent his school time. At that time, the faculty of school was very competent, among which almost all teachers were university graduates and some even returned from overseas. As a boarding school, the atmosphere of teaching and studying was remarkable due to its strict management policy. Therefore, under such circumstances, Yibing Shen cultivated his demanding and scrupulous habit, which provided adequate preparations for his university study and life.

In September, 1958, he was enrolled with excellent admission examination scores by Mathematics Department of Zhejiang Normal College. The chairperson of the department then was Professor Ruiyun Xu, a woman mathematician who graduated from Germany. There were young faculty members such as Silei Wang, Tingfan Xie, Chuanrong Lu and Chuanfang Wang, etc. Some years later, Zhejiang Normal College merged into newly established Hangzhou University, where Professor Jiangong Chen, a famous Chinese mathematician specialized in trigonometric series was the Vice president and Professor Zhengguo Bai was the Chairman of Mathematics Department. During Shen's college years, China was experiencing Great Leap Forward, so students had to switch between a week's learning and a week's labor. Nevertheless, led by Professors Jiangong Chen and Zhengguo Bai, Mathematics Department made great development in both teaching and research. In 1962, with the approval of State Council of China, Professor Chen and Professor Bai began to recruit diplomaed graduate students. It was in the same year when Yibing Shen became one of Professor Bai's five graduate students with his outstanding study records and the result of Graduate Admission Test. During that time, the lack of materials was the biggest problem of doing research. In their graduate years, the five students spent their vacations time in the reference room of Mathematics Department of Fudan University to transcribe research papers and shared them. Although it was tough in every aspect, the department of mathematics provided graduate students with very good supplies. Graduate students of Geometry and Theory of Functions each had an independent office. And students themselves were very hard-working, and they always held discussions in the office. With Professor Bai's strict training, Shen laid a solid foundation for local calculation of differential geometry, especially of riemannian geometry.

Shen graduated in 1965 and got a position to teach at the university. During those ten years of colleges and graduate school, his classmates, colleagues and he always went to factories and the countryside for manual labor. Since then, he realized the importance of combining knowledge at the school with practice.

After the “Cultural Revolution”, Professor Bai recovered diplomaed graduate students recruitment in 1978. Shen as well as more than other 10 graduates and young teachers joined in a seminar class of differential geometry led by Professor Bai. In 1979, besides his differential geometry courses in Hangzhou University, Shen was invited to Mathematics Department of Zhejiang University to teach differential geometry for undergraduate students. The world famous mathematician Fanghua Lin, a professor at Courant Institute, New York, was one of his students in the class that time. In 1980, Shen attended the First Double-Differential (Differential Geometry and Differential Equation) Conference held in Beijing, where he met Professor Shiingshen Chern, an internationally renowned master of differential geometry. Chern is one of Shen’s most admired persons, who amiably called Shen “young hometown-man”. Thereafter, the friendship between the “young hometown-man” and the master lasted more than 20 years till Mr. Chern passed away.

In 1982, when Xiaohuan Mo and Zhenqi Li enrolled, Shen and Professor Bai supervised graduate students together. At the same time, Shen was also the supervisor of Guozhen Lu, a young teacher who had just graduated from the university. He was admitted by Shen and Professor Bai to the Master program in 1986, then he went abroad soon after for the further study. Lu is now a professor of Mathematics Department of Wayne State University, USA and a Distinguished Professor at Beijing Normal University. In 1984, Shenchun Lin and Guofang Wang was enrolled to study differential geometry. Under the supervision of Professors Shen and Bai. Wang is now a mathematical professor of University of Freiburg, Germany. After the “Cultural Revolution”, Professor Bai led his students to transfer research focus from the local Riemannian geometry to the global Riemannian geometry. In 1985, Shen went abroad for the first time to visit the world mathematics center at that time—the Mathematics Department of UC San Diego. Professor Shing-Tung Yau, the young Fields medalist, was hosting a summer school in Differential Geometry and Geometric Analysis, which benefited Shen a lot. Two months later he went to San Francisco, where he met Professor Shiingshen Chern again. Afterwards, he returned to China together with Chern. In China, Chern invited Shen to Tianjin to attend the Special Year in Geometry, mainly focusing on gradient estimates and analysis on Riemannian manifolds. At that time, two articles greatly influenced Shen. One was Chern’s lecture notes on *Minimal Submanifolds in a Riemannian Manifold*, at Kansas University; another was Yau’s early paper *Submanifolds with Constant mean curvature I, II*, published in Amer. J. Math. Shen taught Xiaohuan Mo and Zhenqi Li personally the moving frame method and geometry of submanifold.

Shen had a habit to write handouts himself and then lectured them to graduate students. He also edited teaching materials, lecturing geodesic theory and analysis on manifolds. Not all students in his classes were studying geometry. Some were studying theory of function and other fields, such as Jiecheng Chen, Jiayu Li, etc. Based on his lecture materials, Shen integrated it to a textbook, *Introduction to Riemannian Geometry*, which was in mimeograph form, used by senior undergraduates and junior graduates of Mathematics Department of Hangzhou University. The book deals with the three methods of studying riemannian geometry and in the meantime, gives consideration to classic theories and contemporary development, enabling students to do elementary research by themselves after reading it. It won popularity among graduate students and beginners at many universities in China. With Professor Zhengguo Bai's guide, the book was first published by Higher Education Press after being collaborated by Yibing Shen, Naixiang Shui and Xiaoying Guo. It was revised and published again in 2004 and won the Second Prize for Excellent Textbook of National Institute of Higher Learning awarded by National Educational Committee of China. It was the teaching book or reference book for Graduate School of Chinese Academy of Sciences, Fudan University, Peking University, East China Normal University, Tongji University and many others.

In 1987, Shen was promoted to the rank of full professor. He had the privilege to enjoy the special allowance of the State Council of China and was authorized to be a doctoral supervisor by Academic Degree Committee of the State Council of China in 1993. In 1987, Professors Zhengguo Bai and Yibing Shen won the First Prize for Natural Science; in 1996, Hongwei Xu and Professor Shen won the First Prize for Advance in Science and Technology. Both prizes were awarded by Zhejiang provincial Education Committee for their achievements. In 1998, he won the Third Prize for Advance in Science and Technology awarded by National Ministry of Education. More than 100 of his academic papers have been published in important magazines both at home and abroad, such as *Amer. J. Math.*, *Trans. Amer. Math. Soc.*, *Intern. J. Math.*, *Science in China*, *Acta Math. Sinica* and *Chin. Ann. Math.* He has successively finished standard and key Projects sponsored by National Natural Science Foundation of China, Foundation Projects of Doctoral Program of the Ministry of Education, Foundation Projects of Zhejiang Natural Science and other researches. His achievements mainly are in submanifolds, harmonic maps, integrable systems and real and complex Finsler geometry. In submanifold, he was the first one who improved J. Simons' famous intrinsic rigidity theorem about the scalar curvature of minimal submanifolds in a sphere. In the paper by Shen and Xiaohua Zhu published in *Amer. J. Math.*, they proved a Bernstein-typed theorem about minimal hypersurfaces of finite total scalar curvature in Euclidean space by means of the interior curvature estimate and Gromov's compactness theorem. As to harmonic maps and integrable systems, Uhlenbeck brought in the concept of uniton to the harmonic maps from a two dimensional

domian into a unitary group. In 1996, Yuxin Dong and Shen proved the conjecture about unitons proposed by Uhlenbeck in 1985. Shen and others studied the harmonic maps with finite uniton numbers from a two dimensional simply connected domain to a unitary group, a symplectic group, a complex Grassman manifold and a space form. They therefore proved such harmonic maps could be realized by a finite products of the mappings constructed from the singular Darboux transformations. In Finsler geometry, Shen and his students first studied the geometric variational problems of Riemann-Finsler geometry, working out the Euler-Lagrange equation of Einstein-Hilbert functional in Finsler geometry, and then they studied the first and second variations of harmonic maps between Finsler manifolds. They further studied some existence problems of harmonic maps from complex Finsler manifolds to Kähler manifolds. What's more, Shen and Bang Chen also proved that the Kähler-Finsler metric of a complex Finsler manifold is actually strongly Kählerian, so there are only two kinds of Kähler-Finsler metrics in complex Finsler Geometry. All these results are significant in their own fields.

From 1982, Shen and Professor Zhengguo Bai supervised Master graduate and Ph.D. students. Yuxin Dong, who graduated in 1990, was their first doctoral student. He is now professor in Mathematics Department of Fudan University. Professors Bai and Shen have instructed 46 graduate students and 36 Ph.D. students by July, 2011. Most of those students have become professors or doctoral supervisors in famous universities both at home and abroad, becoming a significant group in the area of differential geometry in China. Take some for example: Xiaohua Zhu, who graduated in 1995, is a professor at Peking University with a title of Changjiang Distinguished Professor of the Ministry of Education. Jixiang Fu, graduated in 1997, is a professor in Fudan University. He was an invited 45-minute speaker at the ICM in 2010 and also the winner of National Outstanding Youth Scholars.

Yibing Shen kept teaching basic courses for undergraduates, from juniors' "Analytic Geometry" and "Differential Geometry" to seniors' "Global Differential Geometry" and "Riemannian Geometry". He has instructed 22 undergraduate theses. He always explained the profound theory in simple terms vividly and stimulatingly, therefore, students all adored his courses. The teaching book he edited for undergraduate students, *Introduction to Global Differential Geometry* (1998, first edition of Zhejiang University Press, 2005, second edition; 2009, third edition of Higher Education Press), covers a complete and systematic introduction to the curve and the curved surface of three dimensional Euclidean space as well as global geometry of the hypersurfaces in higher dimensional Euclidean spaces. Geometry master Professor Shiingshen Chern and Hesheng Hu Academician of Fudan University thought highly of it. Taking the teaching requirements of undergraduate students into account, Shen, together with Weimin Sheng, Xi Zhang and Qiaoling Xia, edited basic materials *Analytic Geometry* (2008, edition of Zhejiang University). In 2006, Shen won University Teaching Master Award of

Zhejiang Province.

In 2010, at time he was 70, Shen retired. In the same year, he was re-employed and continued to teach basic courses for Shing-Tung Yau Elite Class. He also worked as supervisor for differential geometry doctoral and master graduate students, proceeding with talents cultivation and contributing his extra energy to the further development of differential geometry in China.

Weimin Sheng and Guozhen Lu

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