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## Advanced Lectures in Mathematics Volume XXII

# Differential Geometry Under the Influence of S.-S. Chern

edited by

Yibing Shen · Zhongmin Shen · Shing-Tung Yau





Advanced Lectures in Mathematics, Volume XXII Differential Geometry: Under the Influence of S.-S. Chern

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