Symmetry:
A Multi-Disciplinary Perspective

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Lecture Notes Series

Volume 16

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Volume editor

Inder Bir S. Passi
Indian Institute of Science Education and Research

International Press
www.intlpress.com

Ramanujan Mathematical Society
Lecture Notes Series, Volume 16
Symmetry: A Multi-Disciplinary Perspective

Volume editor:
Inder Bir S. Passi (Indian Institute of Science Education and Research, Mohali)

Published in 2012 by International Press, Somerville, Massachusetts, U.S.A. under license from the Ramanujan Mathematical Society.

This work is also published in the Republic of India, exclusively by the Ramanujan Mathematical Society.

ISBN: 978-1-57146-247-3

Printed in the United States of America.

16 15 14 13 12 1 2 3 4 5 6 7 8 9

Preface

The notion of symmetry is fundamental in all sciences, and indeed in all human intellectual endeavors. Its essence is captured through an abstract mathematical treatment based on the Theory of Groups. A Symposium with focus on the notion of symmetry as developed from a mathematical perspective and its interconnections with various other disciplines was held at the transit campus of the Institute of Science Education and Research Mohali located at MGSIPA Complex Sector 26 Chandigarh 160019 during 19–20 February 2010.

The Symposium was motivated by the fact that MTH 101: Symmetry is a mandatory course in the first year for all students of BS-MS at IISERM. While the faculty was fully convinced, the students often wondered about the need and justification for such a course in the very first year of their study. In order to enable the students to see the ubiquity of symmetry across various disciplines, experts from diverse fields were invited to the Symposium which was attended by the IISERM faculty and students.

With a view to make the deliberations at the Symposium available to a wider audience, it was decided to publish the proceedings as a volume which can serve as a supplementary reading for science students at undergraduate and post-graduate levels. It is expected that the articles by the leading experts in their respective fields will serve as a basis for designing instructional material aimed at exhibiting the unity of science. Perceptive readers will find numerous ideas which can be pursued as research projects.

The Organizing Committee is grateful to all the speakers and participants for accepting the invitation to participate, and for contributing their respective articles to this publication.

The Editorial Committee is thankful to the Ramanujan Mathematical Society, and, in particular, the Editor-in-Chief, Professor R. S. Kulkarni, for publishing the proceedings in the Lecture Notes Series in Mathematics.

Inder Bir S. Passi
IISER Mohali

Members of the Advisory Board

Foreword

Symmetry and Symmetry Breaking

When IISER Mohali was set up, there were no departments, schools or centres. The faculty were appointed in various disciplines. Sometimes jointly between two disciplines. Therefore, it was but natural that Professor Passi suggested that we have the first symposium in the Institute and that it be on symmetry. All of us agreed readily, as symmetry pervades all disciplines, living and nonliving, including mathematics and arts and architecture.

As a chemist, I see symmetry everywhere. Chemists classify molecules by symmetry groups. They classify solids by symmetry. More recently, they have started classifying atomic and molecular clusters by symmetry. They use point groups, translational groups and permutation groups as well. They work out selection rules based on symmetry as well as asymmetry. They are obsessed with symmetry as well asymmetry. Asymmetry does not mean not symmetric. It refers to two molecules which are mirror images but are not super imposable. Like the two hands of an individual. Chemistry is dynamic. Naturally, it deals with dynamical systems in which static symmetries are not adequate. One has to deal with dynamical symmetries.

Lots of things around us exhibit symmetry. But on close examination, one realises that the real beauty lies in breaking the symmetry. While the left side of the body looks like a mirror image of the right side, one realises that it is not one hundred per cent true. A chicken can be modelled as a sphere in the first approximation. But to represent a chicken, you need to destroy the spherical symmetry and build things around it!

Life is all about symmetry breaking. When one examines how life must have emerged, it becomes clear that asymmetric molecules must have played a crucial role. Sugars and all the naturally occurring amino acids except glycine are chiral (optically active, that is). Interestingly, the amino acids are of L type (levorotatory). The proteins that are formed out of these L-amino acids are helical and optically active.

How does asymmetry arise from symmetry? When molecules come together and form a crystal, they assemble in a particular way. Quartz is a classic example. Tartaric acid is another. Louis Pasteur could pick up crystals of different optical isomers of tartaric acid using tweezers.

It is known that hexagons can be combined to form a sheet as it happens in a graphite layer. But if you want to produce a football like structure as in fullerene, one needs to destroy the hexagonal symmetry and introduce a pentagon. Around a pentagon, hexagons can be put together to form a bowl. Once you form a bowl, you can add more pentagons and hexagons and produce a ball. Interestingly, that ball is not completely spherical. When Buckminster Fuller proposed the dome structure, nobody would have thought that the chemists would make molecules of that shape one day and start discussions on modifying their properties!

In this little note that I have prepared, I realise that I have touched upon physics, chemistry, biology and architecture, all in one go, by using symmetry and symmetry breaking.

I congratulate Professor Passi and his editorial team in putting together this volume on symmetry, the subject that cuts across disciplines and is vital for our understanding the world around us. I am grateful to all the authors for their valuable contributions. I am particularly happy that it has come from IISER Mohali showing the way for more volumes to come in the near future.

N. Sathyamurthy
Director
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Participants

Faculty and students at IISER Mohali together with the guest speakers participated in the symposium. A special session for discussion and interaction with the students was organized.

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