

Surveys in Differential Geometry

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Volume XVII

Surveys in
Differential Geometry

In Memory of C. C. Hsiung

Lectures given at the JDG Symposium,
Lehigh University, June 2010

edited by

Huai-Dong Cao
and Shing-Tung Yau

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Preface

On May 6, 2009, Professor Chuan-Chih Hsiung passed away. I was given this sad news by his daughter Nancy.

Professor Hsiung had joined Lehigh University in 1952. He officially retired in 1984 but thereafter remained active mathematically—and within the department—for many years.

Besides his substantial and important works in the subject of differential geometry, Professor Hsiung made a tremendously important contribution to mathematics by founding the *Journal of Differential Geometry* in 1967. At that time, he had the foresight and vision to understand that the subject of geometry would grow and become one of the most crucial elements in the development of mathematics.

Note that prior to this no journal has been dedicated exclusively to geometry. Professor Hsiung had the skill to persuade the provost of Lehigh of the journal's importance to the University and to gain his support for the journal. Professor Hsiung set a high standard for the *Journal of Differential Geometry* by inviting first-class mathematicians to its editorial board and by selecting only the best papers.

In the early days of the journal, Professor Hsiung consulted with my thesis advisor, Professor Shiing-Shen Chern, whose advice regarding the journal's development proved invaluable. In 1980 Professors Chern and Hsiung together convinced me to become the journal's editor. We saw the development of geometry influenced by many of the papers which thereafter appeared in the journal.

My collaboration with Professor Hsiung over thirty years' time was pleasant and productive, and I am sad to have lost such a good friend.

I am grateful to Lehigh University for giving this conference, the International Symposium on Geometry and Topology (Bethlehem, Penn., May 2010), in honor of Professor Hsiung. And I am grateful to the many distinguished geometers who presented talks at the conference, and who contributed articles to this volume of the *Surveys in Differential Geometry* book series.

In this volume, we have articles in geometric analysis by Christian Bär and Werner Ballmann on boundary value problems for elliptic differential operators of the first order; by Simon Brendle on rigidity phenomena involving scalar curvature; by Simon Donaldson on stability, birational transformations, and the Kähler-Einstein problem; by D. H. Phong, Jian Song, and Jacob Sturm concerning complex Monge-Ampère equations; by Weimin Sheng, Neil S. Trudinger, and Xu-Jia Wang on the k -Yamabe problem; by S.-T. Yau on topics of geometric analysis. We also have articles in geometry by Gergely Berczi and Frances Kirwan on a geometric construction for invariant jet differentials; by B. Bramham and H. Hofer concerning symplectic dynamics; by Dominic Joyce on C-infinity schemes and C-infinity algebraic geometry; by Kenji Fukaya, Yong-Geun Oh, Hiroshi Ohta, Kaoru Ono on Lagrangian Floer theory on compact toric manifolds; and by Paul Seidel concerning Fukaya categories.

Ten years ago, Huai-Dong Cao came to Lehigh University to lead the geometry group there. He certainly has been the key to managing the *Journal of Differential Geometry*. I am grateful to him for organizing the conference and for editing this volume.

Shing-Tung Yau
Harvard University

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