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Surveys in Geometric Analysis and Relativity

edited by

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ADVANCED LECTURES IN MATHEMATICS

to Richard Schoen
in honor of his sixtieth birthday

Preface

This volume of 23 survey articles is dedicated to Richard M. Schoen on the occasion of his 60th birthday in recognition of his many important contributions as a leading researcher in geometric analysis and general relativity. We also thank him for the equally important roles he has played as mentor, colleague, collaborator, and friend.

Rick Schoen was born on October 23, 1950 in Celina, Ohio. In 1972 he graduated summa cum laude from the University of Dayton and received an NSF Graduate Fellowship. In March 1977, Rick received his Ph.D. from Stanford University under the direction of Leon Simon and Shing-Tung Yau, and soon after received a Sloan Postdoctoral Fellowship. His early work was on minimal surfaces and harmonic maps. By the time that Rick received his Ph.D., he had already proven major results, including his 1975 curvature estimates paper with Simon and Yau.

In the late 1970's, Schoen and Yau developed new tools to study the topological implications of positive scalar curvature. This work grew out of their study of stable minimal surfaces, eventually leading to their proof of the positive mass theorem in 1979. All together, their work was impressive for the way it connected neighboring fields, first using analysis to understand geometry, and then using geometry to understand physics.

In the early 1980's, Rick published a number of fundamental papers on minimal surfaces and harmonic maps. His work on minimal surfaces includes an influential Bernstein theorem for stable minimal surfaces with Doris Fischer-Colbrie. Rick met his future wife Doris in Berkeley, where Doris received her Ph.D. in 1978. They have two children, Alan and Lucy, seen in the photographs in this book, both of whom graduated from Stanford.

Other works from the early 1980's include an extremely useful curvature estimate for stable surfaces, a uniqueness theorem for the catenoid, and a partial regularity theory for stable hypersurfaces in high dimensions with Leon Simon. In 1982, Rick and Karen Uhlenbeck proved the partial regularity of energy minimizing harmonic maps. In 1983, Rick was awarded the very prestigious MacArthur Prize Fellowship.

Rick is also very well known for his celebrated solution to the remaining cases of the Yamabe problem in 1984, this time using a theorem from physics, namely the positive mass theorem, to solve a famous problem in geometry. The resulting fundamental theorem in geometry, that every smooth Riemannian metric on a closed manifold admits a conformal metric of constant scalar curvature, had been

open since the 60's. This work was cited in 1989 when Rick received the Bocher prize of the American Mathematical Society. His work on scalar curvature at this time set the direction for the field for the next 25 years.

Rick was elected to the American Academy of Arts and Sciences in 1988 and the National Academy of Sciences in 1991. He has been a Fellow of the American Association for the Advancement of Science since 1995 and won a Guggenheim Fellowship in 1996.

Starting around 1990, Rick began two major programs. The first was to develop a theory of harmonic maps with singular targets, starting with a joint paper with Mikhail Gromov where they used harmonic maps to establish p-adic superrigidity for lattices in groups of rank one. In a series of papers, Rick and Nick Korevaar laid the foundations for a general theory of mappings to NPC spaces, established the basic existence and regularity results, and applied their theory to settle problems in a number of areas of mathematics. The second big program was a variational theory of Lagrangian submanifolds, including the existence and regularity theory, done in a series of papers with Jon Wolfson.

Over the last decade, Rick has continued to make major contributions to geometric analysis and general relativity. Among other results in general relativity, Rick has made fundamental contributions to the constraint equations (with Corvino and others) which dictate the range of possible initial conditions for a spacetime and proved theorems on the topology of higher dimensional black holes (with Galloway). In geometric analysis, he has several important results with Simon Brendle on Ricci flow, including the proof of the differentiable sphere theorem, as well as a compactness theorem for the Yamabe equation with Marcus Khuri and Fernando Marques.

Rick has written 2 books and roughly 80 papers and has solved an impressively wide variety of major problems and conjectures. He has supervised 35 students and counting, and he has hosted many postdocs. Even with his great success, Rick is still one of the hardest working people in mathematics, giving us all the distinct impression that he must love it. His impact on mathematics, both in terms of his ideas and the example he sets, continues to be tremendous.

We would like to thank all of the authors for their contributions, the publishers Lizhen Ji and Liping Wang for their help, as well as Jaigyoung Choe, Michael Eichmair, John Rawnsley, Peter Topping, and Doris Fischer-Colbrie for contributing photographs. We hope you enjoy reading the beautiful survey articles included in this volume as much as we have enjoyed helping to put it all together.

Hubert L. Bray and William P. Minicozzi II
April 20, 2011

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