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Volume XIX

Arithmetic Geometry and Automorphic Forms

edited by

James Cogdell · Jens Funke · Michael Rapoport · Tonghai Yang

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Arithmetic Geometry and Automorphic Forms

Volume Editors:

James Cogdell, Ohio State University at Columbus

Jens Funke, Durham University

Michael Rapoport, Universität Bonn

Tonghai Yang, University of Wisconsin at Madison

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to Stephen S. Kudla
in honor of his sixtieth birthday

Preface

Stephen S. Kudla turned 60 at the end of August of 2010. We had hoped to mark the occasion with a conference highlighting Kudla's contributions to arithmetic, geometry and automorphic forms. However, Kudla declined the honor ... he says he much prefers the back row to the front row. In its stead, we decided to produce a volume in his honor marking the occasion. We invited 17 mathematicians with personal and/or mathematical connections with Kudla to contribute. The result is the volume you are reading now, the proceedings of a conference that didn't happen.

Kudla has contributed significantly to the themes of arithmetic, geometry and automorphic forms. After writing a thesis on "Real points on algebraic varieties defined by quaternion algebras" under the direction of M. Kuga at Stonybrook, Kudla's interest immediately turned to the arithmetic theory of theta series and through it the Weil representation and the arithmetic theory of automorphic forms. His work to date is bookended by his interest in the geometry and arithmetic of the special cycles occurring in the Fourier coefficients of automorphic forms. He began with the geometry of special cycles that arise as the Fourier coefficients of theta series, particularly the papers with John Millson in the 1970–1980's, and recently (at least since his 1997 *Annals* paper) he has turned to arithmetic algebraic geometry and arithmetic intersection numbers of the cycles that occur in the coefficients of Eisenstein series and their derivatives. In between are the series of work with Rallis on Siegel-Weil formulas and its applications, the work with Harris and Gross on special values of L -functions, as well as the formulation of see-saw dual pairs and theta and epsilon dichotomy in the theory of the theta correspondence. His work on the arithmetic Siegel-Weil formula, and particularly the outline and conjectures that he made in his survey article "Special cycles and the derivatives of Eisenstein series" from the 2001 MSRI workshop Heegner Points and Rankin L -Series, has led to what is now called "the Kudla Program".

Kudla has also contributed significantly to the mathematical community as a teacher and mentor. In his thirty-year career at the University of Maryland he directed 15 PhD theses and is continuing to direct theses at the University of Toronto. He has helped to organize many workshops and conferences over his career, from the 1990 Maryland Conference on the Representation Theory of p -adic Groups to the recent 2008 Thematic Program on Arithmetic Geometry, Hyperbolic Geometry, and Related Topics at the Fields Institute. Notable were the regular series of Oberwolfach meetings on automorphic forms and connections with arithmetic and geometry that he organized with Schwermer and several similar programs at the

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ESI in Vienna. He has been a regular invited speaker at instructional workshops for graduate students and young researchers. His “Schloß Hirschberg” Notes on the Local Theta Correspondence, from his instructional lectures at the European School on Group Theory in 1996, continue to serve as a welcome introduction to the subject. Kudla is a very conscientious professional, as an editor, as a referee, and as a panel member; he has very solid judgments which the community highly values.

As might be expected, the contributions to this volume echo the mathematical interests of Kudla and provide a current snapshot of developments in these areas. The paper of Funke and Millson presents the current state of affairs in the study of special cycles that originated in the late 1970’s, while the contributions of Bruinier, Howard and Yang reflect Kudla’s more recent impact on arithmetic algebraic geometry and intersection theory. Gan’s paper is a recent development on the Siegel-Weil formula with its origins in Kudla’s work with Rallis, whereas the paper of Mœglin takes off from the local Siegel-Weil formula. The paper of Harris, Li and Sun is partially an outgrowth of Harris’ earlier work with Kudla on theta dichotomy. The other papers, those by Cogdell, Ginzburg, Grbac, Gross, Jiang, Jorgenson, Kramer, Rallis, Roberts, Schmidt, Soudry, Schwermer, Vigneras and Wallach, all reflect Kudla’s general interest in the arithmetic of automorphic forms and their L -functions.

We would like to thank all those who made this volume possible. Firstly we would like to thank all the contributors for allowing us to publish their papers. We also thank the (anonymous) referees who are necessary to guarantee the quality of such a volume. We would also like to thank Peng Li at Higher Education Press and Lizhen Ji, who is on the Editorial Board for the *Advanced Lectures in Mathematics* series, for their help with the production of this volume.

Most importantly, we would like to thank Stephen S. Kudla for his contributions to mathematics and the mathematical community. We wish him continued success in the future.

James Cogdell
Jens Funke
Michael Rapoport
Tonghai Yang

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