Calculus

A computer algebra approach

Second Edition

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by Iris Anshel and Dorian Goldfeld
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DEDICATION

The authors would like to dedicate this book to their children, Ada and Dahlia, who played quietly while their parents worked on the manuscript.

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INTRODUCTION

The evolution of the teaching of calculus is at a critical juncture. For some time there has been an emphasis on the computational aspects of calculus in conjunction with the various applications of the method. It is the advent of highly accessible computer algebra systems (CAS) and various sophisticated calculators which has driven us to reevaluate how calculus should be presented. The use of this book does *not* require a computer laboratory, or even access to an elaborate CAS. A graphing calculator suffices in that the emphasis is not on mass computation or programming, but rather on the understanding of the underlying concepts.

A CAS is a computing device with the following capabilities:

(1) *It is a calculator*, i.e., it can perform arithmetic and compute values of standard functions,

(2) *It has 2–dimensional graphics capabilities,* i.e., it can produce graphic displays of functions of a single variable,

(3) *It can do calculus,* i.e., it can compute derivatives and integrals of functions,

(4) *It can do algebra,* i.e., it can expand and simplify algebraic expressions,

(5) *It can create functions,*

(6) *It has 3–dimensional graphics capabilities,* i.e., it can produce graphic displays of functions of several variables.

In light of the immense capabilities of the CAS we are led us to asking the following questions: (1) what approach should we take to this discipline now that the mechanics of computation have been automated, and (2) how can we incorporate CAS with the classical teaching methods?

It is the authors opinion that the new technology is a moment of opportunity. No one would consider learning, for example, Physics or Biology without working in a laboratory; computation for anyone without a CAS is very limited. Calculus is both an intellectual breakthrough and a powerful tool in research and development (in a variety of fields). To understand it as such (and to apply it as a problem solving tool in modern settings) one must obtain both an understanding of the vital concepts in their abstract form and in a wide range of examples. Our philosophy is to focus on the meaning of the most important definitions and theorems and to *experiment* with them on the CAS.

It should be noted that this text is not a "how–to" manual for a particular CAS. Our aim is to teach the concepts of calculus without getting lost in the quagmire of programming. Almost all the exercises and examples can be done at the level of using the CAS as a calculator. Beyond this, even on the most sophisticated CAS, only a few commands need be learned.

Many students are familiar with introductory calculus upon entering college. With this in mind, we have opted to quickly review the foundational material (such as real numbers, functions and their graphs, lines, and circles). Although the introductory material is covered in relatively little space, almost nothing is assumed on the part of the reader (that is to say, the text is substantially self–contained). When the material becomes less familiar and more complex the sections and chapters become more substantial. Throughout the text we focus on core concepts which are presented from first principles. Once a given concept is introduced we utilize the CAS to rapidly view it in many different contexts. This approach encourages the student to develop *hands–on* experience with Calculus. We have found that such a CAS experience is superior to reading hundreds of worked out examples in a book. Evidence of the benefits of this method were apparent when, in 1993–94, an experimental computer laboratory was set up at Columbia University (in conjunction with a course based on this book).

It is the authors hope that this book is intuitively rigorous and indicates how mathematics is thought about. There is little purpose to memorizing massive numbers of formulae and algorithms (all of which the CAS knows). It is a natural outcome of this pedagogical perspective that solutions of problems are presented analytically, and whenever possible and appropriate, algorithmically. The emphasis is consistently on how to derive the formulae, why the algorithms work, and how the CAS can solve Calculus problems.

This edition begins by covering both differential and integral calculus for functions of one variable, mathematical modeling and optimization, basics of ordinary differential equations, and then moves on to differential calculus for vector valued functions and functions of several variables. Much time is spent on vector geometry, coordinate systems, and two and three dimensional graphical display using the CAS. The latter part of the text includes multiple integration, vector fields and line integrals, surface integrals Stoke's theorem, an overview via differential forms, as well as an introduction to Fourier series including a proof of the Fourier expansion theorem.

This book assumes that students have access to a basic CAS with capabilities (1), (2), and (3) above. Among the many exercises the majority can be solved either by hand or with a basic CAS.