# The Iwasawa theory of totally real fields

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From the workshop on the Iwasawa theory
of totally real fields, held at the Indian
Institute of Technology, Guwahati,
September 2008

*Volume editors*

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# Foreword

A Workshop was held at the Indian Institute of Technology, Guwahati, from September 22 until September 30, 2008, with the goal of presenting as much as possible of the background and underlying ideas behind Wiles' celebrated proof of the "main conjecture" of cyclotomic Iwasawa theory over totally real base fields. This workshop was part of a series of schools and workshops organized in various parts of India each year, called "Advanced Training in Mathematics Schools", which are generously supported by the National Board for Higher Mathematics (NBHM). The Organizing Committee consisted of J. Coates, C. S. Dalawat, A. Saikia, and R. Sujatha, and lectures during the course of the Workshop were given by U. K. Anandavardhanan, D. Banerjee, J. Coates, C. S. Dalawat, Narasimha Kumar, E. Ghate, F. A. E. Nuccio, A. Saikia, T. Schmidt, A. C. Sharma, R. Sujatha and O. Venjakob.

We owe to Iwasawa the discovery of the first "main conjecture" in his revolutionary work on cyclotomic fields in the 1960's. He only considered this "main conjecture" for the Tate motive over the field obtained by adjoining all $p$-power roots of unity to the rational field $\mathbb{Q}$, where $p$ is any prime number. However, it was quickly realized that similar ideas could be applied to elliptic curves, and provided a systematic framework for studying the conjecture of Birch and Swinnerton-Dyer, and other conjectural exact formulae in number theory. Today, we believe, although only fragmentary proofs are known so far, that these "main conjectures" hold in vast generality for all motives over $p$-adic Lie extensions of number fields. The great interest of the method discovered by Wiles for proving the "main conjecture" for Tate motives over any totally real number field is that it combines ideas from the theory of automorphic forms with purely arithmetic ideas in a way that suggests the beginning of some deeper general connections between these two vast citadels of modern number theory.

These notes present written versions of most of the material covered by the lectures in the Workshop, and it is hoped that they will be useful to others who wish to study these important questions. They are aimed very much at the beginner in the subject, and try to minimize wherever possible the background knowledge required.

In conclusion, we wish to warmly thank IIT Guwahati for providing such excellent facilities and such a delightful location for holding the Workshop.

**J. Coates**
**C. S. Dalawat**
**A. Saikia**
**R. Sujatha**

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