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Lecture Notes Series

Volume 11

Perspectives in geometry and topology

Proceedings of the Golden Jubilee
International Workshop/Conference in
Geometry and Topology, held at the Indian
Institute of Technology, Bombay,
August 2007

Volume editors

P. Sankaran A. R. Shastri P. Zvengrowski



Ramanujan Mathematical Society Lecture Notes Series, Volume 11 Perspectives in geometry and topology

Volume editors: P. Sankaran A. R. Shastri P. Zvengrowski

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ISBN: 978-1-57146-218-3

Printed in the United States of America.

15 14 13 12 11 1 2 3 4 5 6 7 8 9

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Preface

This Workshop/Conference took place at the Indian Institute of Technology, Bombay, from August 17–25, 2007, in conjunction with the Golden Jubilee of this Institute. As planned, it consisted of a Workshop from August 17–21, followed by a Conference with a broad theme 'Geometry and Topology', from August 23–25, with a free day on August 22. The Scientific Committee consisted of Professors R. Kulkarni (USA and India), P. Sankaran (India), A. R. Shastri (Principal Organizer, India), and P. Zvengrowski (Canada). One of the main goals was to convene a meeting that would be of value to graduate students and younger mathematicians, as well as providing a forum for experienced mathematicians and experts in the field.

The meeting was eminently successful in achieving these goals. The Workshop portion consisted of four minicourses, presented by Professors K. Y. Lam (Univ. Br. Columbia), K. Varadarajan (Univ. Calgary), P. Wong (Bates College, Maine), and P. Zvengrowski (Univ. of Calgary). The format consisted of four 90 minute lectures daily, one for each minicourse, on each of the five days. Each lecturer had ample time to present his material in sufficient detail. Overall the material in these four minicourses encompassed a broad spectrum of important and deep parts of classical and modern algebraic topology, along with related areas in other parts of mathematics such as algebra, geometry, number theory, analysis, and applied mathematics (e.g. the theory of elasticity). It was also remarkable how many interconnections appeared between the four minicourses, as they unfolded. The specific titles of the four minicourses are Topological Methods in the Theory of Bilinear Forms (K. Y. Lam), The Wall Finiteness Obstruction (K. Varadarajan), Combinatorial and Geometric Group Theoretic Methods in Fixed Point Theory (P. Wong), and An Introduction to ℓ_2 -Homology (P. Zvengrowski). Each author has put sufficient effort to make the report up-to-date.

The Conference portion consisted of about seven lectures for each of the three days (total 21), ranging from 45–55 minutes each. The lecturers included some of the most distinguished people in the field. Topics ranged over a broad spectrum, from theoretical physics to topology, geometry, algebra, etc., and included reports about some of the most recent important developments in the field such as the recent solution of the Poincaré Conjecture by Pereleman *et al.*, to name just one example.

This Proceedings consists of complete notes from the four lecture series of the workshop and reports representing a sample of the lectures given in the conference part. In addition to the subjects already mentioned, they also represent the interplay of modern algebraic topology with algebraic geometry, algebraic groups, differential geometry, and non-commutative geometry. We feel that the volume will have a lasting

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value for students and researchers in the specific areas that are covered. Thanks are not only due to the authors, but also to all those who generously helped in the editorial and refereeing work for the various submissions.

One somewhat unique feature of the meeting, that was greatly enjoyed by all, was the almost daily concert of Indian Classical Music, following each session. On the free day between the Workshop and Conference parts, there was a boat trip and visit to the Elefanta Caves and Temples, as well as to downtown Mumbai for a little sight-seeing and shopping.

The Organizers

Foreword

I am happy to place, in the hands of the mathematical community, this volume of the Proceedings of the International Workshop/Conference on Geometry and Topology organised on the eve of the Golden Jubilee celebrations of our institute. We hope that this volume will be of good use to students and experts in this area.

On behalf of the organizers of the event, my sincere thanks are due to all the participants and experts who have contributed to this volume. Thanks are due to my colleagues and the staff at Department of Mathematics, IIT Bombay for the support during the event. Thanks are due to the Ramanujam Mathematical Society (RMS) for coming forward to publish this volume and thanks to the technical staff at RMS for doing an excellent job of editing.

A. R. Shastri Department of Mathematics I.I.T. Bombay

List of Registered Participants

17th August to 25th August 2007

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