

Surveys in Differential Geometry

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Volume V

Surveys in
Differential Geometry

Differential geometry
inspired by string theory

edited by
Shing-Tung Yau



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Differential geometry inspired by string theory

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Introduction

In this volume of Surveys in Differential Geometry, we collect papers on the part of geometry that is related to the modern development of string theory. Recall that in the first volume of Surveys in Differential Geometry, we have the paper by Witten on strings with gravity which was applied to calculate chern numbers for certain line bundles over moduli space of curves. The geometric ideas initiated by consideration of string theory have been tremendous successful, and we believe that it is time to collect articles that can survey the subject in a reasonable way. The papers in this volume only represent some part of the interaction between string theory and geometry that is important and rigorous.

We have the paper by Bismuit and Labourie on the Verlinde formula, which is an accumulation of ideas from physicists and mathematicians. We have the paper of Aspinwall on K3 surface and string duality, which is a simplest nontrivial space to test some very nontrivial duality theory in string theory. We have the paper by Bryan and Leung on counting curves on surfaces, which is also inspired by works on branes in string theory. We have the paper of Kefeng Liu on how to apply ideas of localization through heat kernel to various geometric model, which in particular inspired the previous mentioned work of Bismuit and Labourie. Finally, we have collected a few papers related to mirror symmetry among Calabi-Yau manifolds. While there have been tremendous activities in this subject in the past ten years, the final rigorous treatment of the formula by Candelas and others on toric varieties is only accomplished in the paper of Lian-Liu-Yau, which we reproduce here. Li and Tian's paper surveys both quantum cohomology and the virtual cycles which are part of the whole theory. In 1996, Strominger-Yau-Zaslow come up with a geometric construction of mirror manifolds, which needs a lot more rigorous mathematical treatment. Mark Gross's paper represents a good survey in the topological and some geometrical side of this.

We hope these papers will provide a good starting point for geometers who may want to get into the subject of the interaction of geometric and string theory.

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