Seiberg-Witten and Gromov invariants for symplectic 4-manifolds

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edited by
Richard Wentworth

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by Clifford Henry Taubes (Harvard University)

Richard Wentworth, editor

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Preface

On March 28-30, 1996, International Press, the National Science Foundation, and the University of California at Irvine sponsored the First Annual International Press Lecture Series, held on the Irvine campus. The inaugural speaker for this event was Professor Cliff Taubes of Harvard University who delivered three lectures on "Seiberg-Witten and Gromov Invariants." In addition, there were ten one-hour lectures delivered by some of the foremost researchers in the field of four dimensional smooth and symplectic topology. Volume I of these proceedings contains articles based on six of those lectures.

The present volume consists of four papers by Taubes comprising the complete proof of his remarkable result relating the Seiberg-Witten and Gromov invariants of symplectic four manifolds. The first paper "SW $\Rightarrow$ Gr: From the Seiberg-Witten equations to pseudo-holomorphic curves" appeared in print in 1996 in the Journal of the American Mathematical Society. The remaining three papers appeared in the Journal of Differential Geometry. See the references below.*

It is especially gratifying to have this magnificent work collected together in one volume. Thanks are due to Cliff Taubes for agreeing to this arrangement, to the aforementioned journals for their permission to reprint the first two articles, and to Hugh Rutledge of International Press for his efforts in making this possible.

The idea for a distinguished lecture series was conceived by Peter Li and S.-T. Yau. The first conference was organized by Ronald J. Stern and Richard Wentworth, with the important assistance of Julie Crosby and the Department of Mathematics at UC-Irvine. We would like to thank the invited speakers and the audience of over 200 research mathematicians whose participation made this conference a great success.

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2. *Counting pseudo-holomorphic submanifolds in dimension* 4, J. Differential Geom., **44** (1996), 818–893.

3. Gr $\Rightarrow$ SW : *From pseudo-holomorphic curves to the Seiberg-Witten invariants*, J. Differential Geom., **51**(**2**) (1999), 203–334.

4. SW = Gr : *Counting curves and connections*, J. Differential Geom., **52**(**3**) (1999), 453–609.

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