Teichmüller theory and moduli problems

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Volume editors

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Introduction

The Harish-Chandra Research Institute (HRI), Allahabad, India, hosted the "US-India Workshop: Teichmüller Theory and Moduli Problems" from January 5, 2006 to January 15, 2006. This was the culmination of the "Year in Teichmüller Theory and Moduli Problems." This year-long program was inaugurated in January 2005 by Clifford J. Earle (Cornell University, USA), and Frederick P. Gardiner (City University of New York, USA). They visited HRI during January – February, 2005, and gave a series of introductory lectures on Teichmüller theory. Clifford Earle gave a series of lectures on "Teichmüller theory, past, present, and future." Frederick Gardiner gave some lectures on "A short course on Teichmüller's theorem." These lectures were followed by some survey lectures on "Moduli spaces of vector bundles" by M. S. Narasimhan in March 2005.. William J. Harvey of King's College, London, visited HRI in March 2005 and lectured on "Mapping Class Groups." He surveyed some recent work about these groups which play a pivotal role in low-dimensional topology and geometry of surfaces. John H. Hubbard (Cornell University, USA) visited HRI in August 2005 and gave a course of lectures on "Holomorphic Dynamics."

The principal aim of this Workshop was to bring closer two mathematical traditions, namely the Ahlfors-Bers School of the complex analytic approach to Teichmüller theory that has a strong presence in USA, and the algebro-geometric tradition of the moduli of vector bundles that has developed in India. The idea was to connect scholars from different countries with similar research interests, as an important step towards furthering joint research. The larger objective was to foster increased cooperation between the Indian, American and European scientific communities, working in various areas of moduli problems.

Teichmüller theory originated as a specialized branch of complex analysis. Over the last 30 years, it has blossomed into a big and booming field of research, interacting with many other areas of mathematics, like topology, geometry, and dynamics. A natural question in the study of Riemann surfaces is to parametrize the space of conformal structures on a given topological surface. Riemann mapping theorem already says that there is a unique conformal structure on the Riemann sphere, complex plane and the unit disk, i.e any Riemann surface structure on a simply connected domain is equivalent to one of these three cases. There is a well-studied 1-complex parameter family of Riemann surface structure on a compact surface of genus 1 (elliptic curves).

Riemann asserted that, for a compact Riemann surface of genus $g \geq 2$, the space M_g of distinct conformal structures has complex dimension $3g - 3$. The space M_g is called the Riemann's moduli space. The algebraic geometers have studied this space extensively. During the late 1930s, Teichmüller followed an analytic approach, using quasiconformal mappings, and constructed a new space T_g, now called the Teichmüller space. The Teichmüller space has a canonical complex structure. The automorphism group of this structure is Γ_g, the Teichmüller Modular Group, which is isomorphic the outer automorphism group of the fundamental group of a compact surface of genus g. The quotient space T_g / Γ_g can be canonically identified with M_g.

Broadly speaking, there are three main approaches to the study of moduli of Riemann surfaces and their higher-dimensional analogues:

(i) The analytic approach was initiated by Teichmüller and was further developed by Ahlfors, Bers, and their students and followers. An important aspect of this approach is that it can also be extended to noncompact surfaces including those of infinite type. The methods of quasiconformal mappings and Teichmüller theory also found interesting applications in the study of the dynamics of rational maps. Fundamental work in this area was done by Douady, Hubbard, McMullen, Sullivan, Thurston, and others during the 1980s.
(ii) The algebro-geometric approach was developed by Grothendieck, Deligne, Mumford, and many others.
(iii) The topological and hyperbolic/conformal geometric approach was initiated by Nielsen and Fenchel in the 1930s. Its importance for the study of 3-manifolds and Kleinian groups was recognized by Kulkarni, Marden, Maskit, and was revolutionized by Thurston, and later developed in fundamental ways by McMullen, Penner, Sullivan, and others in the 1980s.

As is well-known, Teichmüller theory has found interesting applications in modern particle physics. In the branch of physics known as String theory, elementary particles are modelled by loops of strings that generate Riemann surfaces as they move through time. These Riemann surfaces have physical interpretations, and Teichmüller theory can be used to study how they vary.

The mapping class group of a genus g surface acts on T_g, and the quotient M_g, as noted before, parametrizes conformal structures on that surface. We can have interesting parameter spaces for many other structures and objects other than the conformal structure. For example, we can have moduli spaces of holomorphic vector bundles on a given base, moduli space of holomorphic maps from surfaces of fixed genus to a given projective variety etc. We refer to the article of Seshadri (in this volume) for a more technical introduction for the moduli spaces.

This workshop was funded by the National Science Foundation, USA, Department of Science and Technology, India, and the Infosys Foundations. There were speakers from USA, UK, Spain, Japan, China, France, Germany, Switzerland, Mexico, Sultanate of Oman, and India. The lectures covered a remarkably broad range, helping the younger participants to get an exposure to current research areas. There was also a special event – John Hubbard (USA), Richard Canary (USA), and

S. Ramanan (India) gave general survey lectures on the historical developments and various ramifications of Teichmüller theory, hyperbolic geometry, 3-manifolds, and moduli of vector bundles.

The editors wish to thank the graduate students and the staff at HRI who worked very hard to make this workshop a success. Many thanks to Professor Amitabha Raychowdhury (Director, HRI), and the graduate students Vikram Aithal, Kuntal Chatterjee, Krishnendu Gangopadhyay, and Siddhartha Sarkar. Special thanks to Prof. Madhav Modak, who took pains to go through every paper, and corrected many typos and ambiguities.

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