# Essays on geometric group theory

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#### Ramanujan Mathematical Society

## Lecture Notes Series

Volume 9

## Essays on geometric group theory

Proceedings of the instructional workshop and international conference on group theory, Indian Institute of Technology, Guwahati, December 2002

Volume editor

N. S. Narasimha Sastry



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Volume Editor: N. S. Narasimha Sastry

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#### **Preface**

Geometric ideas have played a very major role in the development of Group theory in 20th century. Coxeter groups and Coxeter complexes, Tits theory of buildings, Bass-Serre theory of Groups acting on trees, Gromov's notion of hyperbolic groups and combinatorial methods have all been very central in several areas of mathematics.

With the objective of introducing the basic ideas and recent developments in these topics to young researchers in various Research Institutions and Universities in India, the Indian Statistical Institute at its Bangalore Centre and the Indian Institute of Technology, Guwahati, organized "Instructional Workshop and International Conference on Geometric Group Theory" during December 2–21, 2002, at the beautiful campus of the Indian Institute of Technology, Guwahati.

In the Workshop part of this programme (December 2–14, 2002), five short courses of lectures were given. The topics were:

- Coxeter Groups (10 lectures) by R.P. Shukla and Maneesh Thakur
- Buildings and Tits Systems (10 lectures) by N.S. Narasimha Sastry
- Bass-Serre Theory (10 lectures) by A. Raghuram and B. Sury
- Hyperbolic Groups (5 lectures) by A. Naolekar
- Combinatorial Group Theory (5 lectures) by I.B.S. Passi and Roman Mikhailov

In the Conference part of this programme (December 16–21, 2002), experts in this area of mathematics were invited to give research/expository lectures in the topic of their expertise. The articles in this volume are mostly the expanded versions of the lectures delivered by the authors in this Conference. It is hoped that this volume will be of some assistance to mathematicians who would like to familiarize themselves with these topics.

The Organizing Committee for this programme consisted of Professors I.B.S. Passi (Chairman), N.S. Narasimha Sastry, B. Sury, B.K. Sarma and Meenaxi Bhattacharjee.

Apart from the Indian Statistical Institute and the Indian Institute of Technology, Guwahati, the programme also received financial support from the National Board for Higher Mathematics, India; The British Council, India; Harish-Chandra Research Institute, Allahabad; Oil and Natural Gas Commission and Oil India Limited.

I wish to thank all the speakers in this programme and the authors of the articles, without whose enthusiastic support, this programme would not have achieved its objective.

As a member of the organizing committee, I also thank all the sponsors for their financial support; the Indian Institute of Technology, Guwahati, for the excellent facilities and enthusiastic and cheerful support extended during the Conference; and the Ramanujan Mathematical Society for bringing out this volume.

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I also would like to record my appreciation of the enthusiasm, meticulous attention to detail and the caring of Professor Meenaxi Bhattacharjee, her colleagues and students at IIT, Guwahati, which contributed largely towards making this event a very pleasant and fruitful experience. I am confident that all the participants at this event share this sentiment.

Finally, I would like to thank Ms. Asha Lata for her help in compiling this volume.

Date: 5 December 2008 Indian Statistical Institute Bangalore Centre

N. S. Narasimha Sastry Editor

## Talks given in the Conference

Professor Peter Abramenko Some applications of Bass-Serre theory to the  $GE_2$ 

question

Professor Roger M. Bryant Metanilpotent varieties of groups

Professor Marc Burger Tree automorphisms, products of trees and lattices

Professor Alan Camina Block designs

Dr. Indira Chatterji Twisted idempotent conjectures

Dr. Ahmad Erfanian On some groups with the zero presentation ranks

Mr. Shripad M. Garge On conjugacy of Weyl groups

Professor Narain Datt Gupta A group with the slowest sub-exponential growth

Professor C.K. Gupta Recent results on automorphism groups

Professor Alessandra Iozzi Trees and hyperbolic spaces

Dr. D.L. Johnson Word growth of groups

Professor S.S. Khare Group actions on smooth manifolds with specific

fixed point sets

Dr. Linus Kramer Symmetric spaces, asymptotic cones and buildings

Professor Ravi Kulkarni Idea of a branched covering

Mr. Roman Mikhailov On localization in groups and group varieties

Professor Shahar Mozes Tree automorphisms, products of trees and lattices
Professor Himadri Mukherjee On the isomorphism problem and related problems

Professor I.B.S. Passi Centralizers and homology

Dr. Laszlo Pyber Boundedly generated groups

Professor M.S. Raghunathan The word metric on lattices in higher rank semi-

simple Lie groups

Dr. A. Raghuram Depths and conductors of representations of p-adic

groups

Dr. Bertrand Remy
Non-linearities of Kac-Moody groups

Professor Mark Ronan Constructing buildings and twin buildings

Dr. Parameswaran Sankaran On automorphisms and quasi-isometries of finitely

generated groups

Professor Marcus du Sautoy Counting subgroups in nilpotent groups and points

on elliptic curves

Professor Dan Segal Subgroups of finite index in profinite groups

Professor J.B. Srivastava Invariants of certain classical and reductive groups

and their applications in computer vision

Dr. Geetha Venkataraman Finite group enumerations: a brief introduction

Professor D.N. Verma A natural representation model for the symmetric

groups, and the ensuring potential for a new unified

representation theory for Coxeter Groups

Dr. Christopher Voll Zeta functions of groups and enumeration in Bruhat-

Tits buildings of  $Sl_n(Q_p)$ 

Professor John S. Wilson Aspects of growth of groups

## **Conference Participants**

Professor Peter Abramenko

Dr. Meenaxi Bhattacharjee

Professor Roger M. Bryant

Professor Marc Burger

Professor Alan Camina

Dr. Indira Chatterji

Professor B.P. Chetiya

Dr. Ahmad Erfanian

Professor C.K. Gupta

Professor Narain Datt Gupta

Professor Alessandra Iozzi

Dr. D.L. Johnson

Professor S.S. Khare

Dr. Linus Kramer

Professor Ravi Kulkarni

Professor Ramji Lal

Professor Shahar Mozes

Professor H.K. Mukherjee

Professor C. Musili

Dr. A. Naolekar

Professor I.B.S. Passi

Professor Dipendra Prasad

Dr. Laszlo Pyber

Dr. K.N. Raghavan

Professor M.S. Raghunathan

Dr. A. Raghuram

Dr. Bertrand Remy

Professor Mark Ronan

Dr. Paraeswaran Sankaran

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Professor Marcus du Sautoy

Professor Dan Segal

Professor B.L. Sharma

Dr. Ravindra Prasad Shukla

Professor J.B. Srivastava

Dr. B. Sury

Dr. Maneesh Thakur

Professor D.N. Verma

Professor Christopher Voll

Professor John S. Wilson

## **Workshop Participants**

Md. S K Safique Ahmad

Ms. Sasmita Barik

Mr. Dhruba Jyoti Bora

Ms. Ardeline Mary Buhphang

Mr. Anandaram Burhagohain

Mr. Moon Kumar Chetry

Ms. Jonali Choudhury

Mr. Biren Das

Dr. Partha Pratim Dey

Dr. Sanjay Kumar Dutta

Dr. Sanghita Dutta

Mr. Shripad M Garge

Mr. Sahzamal Hoque

Mr. Rafiqul Islam

Mr. Rupesh Kumar Jha

Mr. Bidyut Kalita

Mr. Roman Mikhailov

Mr. Milan Nath

Mr. Tulasi P Nepal

Mr. Binod Kumar Sahoo

Mr. Siddhartha Sarka

Mr. Ritumoni Sarma

Mr. Brajesh Kumar Sharma

Mr. Angom Tiken Singh

Mr. Anupam Kumar Singh

Mr. Ningthoujam Jiban Singh

Mr. Thokchom Vikramjit

Ms. Hiramala Talukdar

Ms. Madhusmita Tripathy

Dr. Geetha Venkataraman

Dr. Christopher Voll

Mr. Akhilesh Chandra Yadav

Mr. Satya Narayan Yadav

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