Number theory and discrete geometry

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Volume 6

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Volume editors

R. Balasubramanian S. G. Dani P. M. Gruber R. J. Hans-Gill

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Volume Editors: R. Balasubramanian S. G. Dani P. M. Gruber R. J. Hans-Gill

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Foreword

When I reflect on the great number theorists of India, the names that come to my mind are S. Ramanujan, S. S. Pillai, S. D. Chowla and R. P. Bambah. Chowla and Bambah have also contributed greatly by building good schools around them. In the context of the general drift towards many newer topics, building up a school in a hard area like classical number theory (which has been laboured on for ages) is indeed a difficult task. It is to the immense credit of Prof. R. P. Bambah to have, in addition to prolific and deep contributions of his own, in the areas of Geometry of Numbers, Packings and Coverings, Discrete Geometry, Diophantine Approximations and classical Number Theory, built such a school of high standard.

Professor Bambah has played a pivotal role in the development of the theory of coverings, and his work has triggered a good deal of further research on the theme. There are many significant results in the area proved by him and by others benefiting from his guidance. Proofs of Watson's conjectures and other related conjectures on non-homogeneous quadratic forms, progress in finite coverings, in the results on covering densities for spheres and the well-known conjecture of Minkowski are some notable examples.

Professor Bambah holds the distinction of having close association with many celebrated mathematicians like H. Davenport, L. J. Mordell, P. Erdös, K. F. Roth, H. Zassenhaus, and so on. He has been very interactive and has had numerous research collaborations, involving a wide spread of mathematicians: S. Chowla, H. Davenport, C. A. Rogers, A. C. Woods, H. Gupta, K. Rogers, K. F. Roth, H. Zassenhaus, V. C. Dumir, R. J. Hans-Gill, N. J. A. Sloane, I. S. Luthar, M. L. Madan, D. D. Joshi, D. B. Lahiri.

Professor Bambah is very erudite on the development of number theory. He is an authority on the history of the Manchester School built by L. J. Mordell and the Cambridge School built by G. H. Hardy and J. E. Littlewood. He is also very knowledgeable about S. Ramanujan, E. C. Titchmarsh, A. E. Ingham and many others.

I would like to recall here a simple looking result proved by Bambah and Chowla which deserves keen attention of the coming generations of mathematicians. Here it is: let q_1, q_2, \ldots be the ascending sequence of natural numbers which are sums of two squares of integers. Then as *n* tends to infinity, we have $q_{n+1} - q_n = O((q_n)^a)$, where $a = \frac{1}{4}$. Bambah and Chowla gave an *O*-constant. It is a great challenge to improve even their *O*-constant (let alone proving $o((q_n)^a)$, with $a = \frac{1}{4}$). It is a still greater challenge to prove the result with *a* arbitrarily small.

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I could go on, writing more and more about Professor Bambah. However I will conclude here, wishing him and his folk (including his family and his students) a long and happy life.

K. Ramachandra

Preface

An International Conference in Number Theory and Discrete Geometry was held at the Department of Mathematics, Panjab University, Chandigarh from November 30 to December 3, 2005, in honour of Professor R. P. Bambah on the occasion of his 80th birthday. This volume represents the proceedings of the conference. Some invited speakers who could not attend the conference have also contributed to the volume.

We thank the authors for their contributions to this volume, and the referees of all papers for helping to improve the quality of the volume. We also take this opportunity to thank all those whose efforts contributed to the success of the conference, including the Organising Committee of the conference, the participants at the conference, and the faculty and administrative staff of the Mathematics Department, Panjab University who enthusiastically carried out various responsibilities for the conference. Thanks are also due to National Board for Higher Mathematics, Council for Scientific and Industrial Research, Indian National Science Academy and Panjab University, Chandigarh for financial support to the conference. We are also thankful to the Ramanujan Mathematical Society for agreeing to publish the proceedings in the RMS Lecture Notes Series sponsored by the Department of Science and Technology.

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