

# Commutative algebra and combinatorics

## Ramanujan Mathematical Society Lectures Notes Series

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*Ramanujan Mathematical Society*

Lecture Notes Series

Volume 4

# Commutative algebra and combinatorics

Proceedings of the international conference  
on computational algebraic geometry,  
Harish-Chandra Research Institute,  
Allahabad, December 2003

*Volume editors*

R. V. Gurjar

S. A. Katre

Ravi A. Rao

J. K. Verma



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Commutative algebra and combinatorics

Volume Editors:

R. V. Gurjar  
S. A. Katre  
Ravi A. Rao  
J. K. Verma

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# Foreword

One of the most useful functions of the International Joint Meetings run by the American Mathematical Society with various countries in recent years is the opportunity for specialized “shoulder” conferences mixing participants from many different countries. Despite the wonders of modern email and the web, anyone who has gone to such an event knows that the level of collaboration and interaction developed in such face-to-face meetings greatly exceeds what is possible by any other means.

Not only do senior researchers from the different countries have opportunities for discovery of mutual interests and the initiation of cooperation, but students can interact with teachers who bring a truly international perspective on the field, one not accessible in any one place. Introductory and survey lectures on a variety of topics of intense current interest bring a broad perspective that can dramatically influence the course of a young person’s work.

The current volume makes available a record of the expository and survey talks at just such a meeting, the International Conference on Commutative Algebra and Combinatorics, organized by the Bhaskaracharya Pratishthana, Pune, and the Harish-Chandra Research Institute, in Allahabad just before the joint meeting in Bangalore, in December 2003. The 5 survey articles and the extended tutorial cover a remarkable range of current computational commutative algebra and algebraic geometry.

The central objects of study could be described with only the slightest oversimplification as ideals of monomials (Stanley-Reisner Theory) and rings generated by monomials (Toric Geometry). There is a clear description of the basic problems and constructions of current interest, as well as some of the major applications of this theory, from the enumeration of “magic squares” to algebraic statistics. Throughout there are plenty of open problems to orient newcomers to the area to the exciting work that is—and will be!—going on. I think that these expositions and surveys will be useful to any student (and lots of mature mathematicians) interested in aspects of these fields.

DAVID EISENBUD  
Berkeley, California  
September 25, 2007

International Workshop on Computational Algebraic Geometry  
and  
International Conference on Commutative Algebra & Combinatorics  
(December 8–13, 2003)

---

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# Preface

On the occasion of the first International Meeting of the American Mathematical Society in India organised by the Indian Academy of Sciences and Indian National Science Academy at Indian Institute of Science, Bangalore from December 17–21, 2003, the Bhaskaracharya Pratishthana felt it would be an ideal opportunity to further its interest in the spread of computational and combinatorial methods in commutative algebra.

The Bhaskaracharya Pratishthana appropriately has taken the lead in developing this frontier area of research in accordance with its traditions begun by its founder, and continued and extended by its present Managing Committee.

The Bhaskaracharya Pratishthana, Pune and Harish-Chandra Research Institute, Allahabad (H.R.I.) jointly organized an international workshop on Computational Algebraic Geometry from December 8–13, 2003 at the Harish-Chandra Research Institute, on the banks of the river Ganga. During the same period an International Conference on Commutative Algebra and Combinatorics was also organised by BP and HRI at the same venue.

An international organising committee was constituted to oversee the academic aspects of the school type workshop planned in the morning sessions accessible to graduate students, and in choosing the speakers for the international conference being planned for the afternoon sessions. In the morning session the participants were exposed to several front-line areas of commutative algebra, and some of the recent developments and open problems in those areas. A computational laboratory was set up at H.R.I. with 15 computers and special free computational software such as *Macaulay 2*, *CoCoa 4.1*, *4ti2*, *CaTS*, *Gfan*, *Polymake*. The computational workshop was funded by a generous grant from the Department of Science and Technology (D.S.T.), Government of India. The international conference was partially supported by the National Board for Higher Mathematics (N.B.H.M.), International Mathematical Union, National Science Foundation, U.S.A., and the organizing institutions.

The international organising committee decided to focus on expository lectures in a few areas of commutative algebra which were prominent today, and to lead to open problems which could be tackled by students. It was also decided to separate the areas of research being covered in this school from those being covered in the session on Commutative Algebra being organised at the A.M.S. meet in Bangalore. There were tutorial sessions in the evenings in order to emphasize the computational aspects and to include some practical training.

The topics and speakers for the morning and afternoon sessions were:

## **Speakers (Expository talks)**

1. Irena Swanson, New Mexico State Univ., USA, Primary Decompositions.
2. N. V. Trung, Institute of Mathematics, Hanoi, Vietnam, Regularity, Castelnuovo-Mumford Regularity and Related Invariants.

3. Serkan Hoşten, San Francisco State University, A Survey of Toric Initial Ideals.
4. Winfried Bruns, University of Osnabrück, Linear Diophantine Equations and the Anand-Gupta-Dumir Conjectures.

### **Speakers (Computational Sessions)**

1. Diane Maclagan, Rutgers University, U.S.A.
2. Rekha Thomas, University of Washington, Seattle, U.S.A.

### **Associates (Practical Sessions on Macaulay 2)**

1. Amit Khetan (UMass, Amherst, U.S.A.)
2. Leah Gold (Texas A&M Univ., U.S.A.)
3. A. V. Jayanthan (I.I.T. Bombay)
4. Tony Puthenpurackal (I.I.T. Bombay)
5. Sara Faridi (Dalhousie University, Canada)

Part I of the present volume is a compilation of the material disseminated in the computational part, under the leadership of Diane Maclagan and Rekha Thomas. The school was based on the book *Gröbner Bases and Convex Polytopes* by Bernd Sturmfels (A.M.S. 1995). The topics in this book provide an ideal combination of combinatorics, algebra and computation. There were six main lectures in the school, one each day, followed by three to four hours of hands on computational exercises using *Macaulay 2* and *CoCoA*. The main topics were as follows: Gröbner basics, The Gröbner fan, toric ideals, triangulations, and minimal free resolutions.

Part II of this volume consists of articles based on the expository talks. This part is edited by W. Bruns and it also contains the notes supplied by T. Hibi, who was unable to attend the conference.

In view of the fact that 2003 was the 60-th year of two of India's major contributors in Commutative Algebra and Combinatorics related to it, viz. R. C. Cowsik and V. C. Dumir, it was felt appropriate to have the international conference on Commutative Algebra and Combinatorics at this juncture, in their honour. This conference was held in parallel to the workshop. Professor V. C. Dumir was felicitated in this conference. Unfortunately, Prof. Dumir passed away all of a sudden on February 3, 2006. This volume is dedicated to his memory.

R. V. GURJAR   S. A. KATRE   RAVI A. RAO   J. K. VERMA



*This volume is dedicated to the memory of*

**Vishwa Chander Dumir**



( 1943 – 2006 )



# Vishwa Chander Dumir

1943–2006

Professor Vishwa Chander Dumir, an eminent mathematician and a distinguished scholar, left for his heavenly abode suddenly, on February 3, 2006.

Professor Dumir was born on June 7, 1943 at Mighiana, district Jhang now in Pakistan. He was the second son of Sh. Bhagwan Dass Dumir and Smt. Parkash Vati. He got his early education at Roorkee (U.P.). In 1953, his father was transferred to Chandigarh and Dr. Dumir had the opportunity to see this beautiful new city grow. He used to speak nostalgically about it. He had a brilliant academic career. He graduated from Government College Chandigarh and stood first in B.A. Honours in Mathematics. He did his M.A. (Mathematics) from Department of Mathematics, Panjab University in 1962 and stood first in the University. He joined the Department of Mathematics, Panjab University, Chandigarh for research and later started working under the guidance of Professor R.P. Bambah and went to Ohio State University, Columbus, Ohio, in 1964 with him, where he was awarded Ph.D. in 1965. The title of his thesis was *Diophantine Inequalities for Quadratic and other Forms*. He served as Assistant Professor at Ohio State University (1966) and at University of Illinois at Urbana-Champaign from 1966 to 1968. He joined the Department of Mathematics, Panjab University as a reader in 1968 in July 1968 and was promoted as a professor in 1979.

Professor Dumir made significant contributions in number theory especially in the fields of geometry of numbers and discrete geometry. Some of his major achievements are: proof of a conjecture of G.L. Watson on non-homogeneous minima of indefinite quadratic forms, proof of a conjecture of Bambah, Dumir and Hans-Gill on positive values of non-homogeneous indefinite quadratic forms, proof of a conjecture of L. Fejes Toth on saturated system of circles and convex domains, proofs of conjectures of Mahler and Jackson. He proved results on lattice double packings and lattice double coverings in the plane, an analogue of a problem of Mordell and general covering density of star domains. He worked extensively on view-obstruction problems, developed a general theory and generalized it to subspaces and flats, obtained several special results on these and on the Billiard Ball motion problem of Schoenberg. He also contributed to the determination of idempotent generators of minimal cyclic codes. He collaborated with Professors R.P. Bambah, H. Gupta, R.J. Hans-Gill, J.B. Wilker, A.C. Woods, D.S. Khassa, V.K. Grover, Madhu Raka and Dr. Ranjeet Sehmi, Dr. Harsh Anand, Dr. Gurmeet Kaur and Ms. Anuradha Sharma.

He was elected Fellow of Indian National Science Academy in 1983 and of the National Academy of Sciences in 1985. He was awarded Srinivasa Ramanujan Birth Centenary Award by Indian Science Congress Association in 2004. He visited Universities of Cambridge, Oxford and London under the British Council Exchange Programme in 1972. He visited University of Illinois in 1977–78 under the Indo-American Fellowship of UGC. He was visiting professor to Ohio State University (1986–88), University of Colorado (1998), University of Illinois (2000–2001). He

delivered invited talks in seminars and conferences at many places in India and abroad. He was honoured on the occasion of his 60th birthday felicitations for his significant contributions to geometry of numbers and discrete mathematics during the International Conference on Commutative Algebra and Combinatorics held at Harish-Chandra Research Institute, Allahabad, December 8–13, 2003. He delivered 16th Srinivasa Ramanujan Memorial award lecture at the annual conference of the Indian Mathematical society held at Roorkee in December 2005.

Professor Dumir has published more than 45 research papers in a variety of areas of his interest. He was an excellent teacher and eminent researcher. As a person, Professor Dumir was very cordial, hospitable and friendly. The Mathematics community has suffered irreparable loss due to his sudden demise. He is survived by his wife, Saroj, and two sons, Sanjeev and Neeraj, both of them are accomplished engineers.

*Rajinder J. Hans-Gill*

# Part I

---

## COMPUTATIONAL ALGEBRA AND COMBINATORICS OF TORIC IDEALS

---

Diane Maclagan

Rekha R. Thomas

*With the co-operation of*

Sara Faridi

Leah Gold

A. V. Jayanthan

Amit Khetan

Tony Puthenpurakal

## List of Participants at the International Workshop on Computational Algebraic Geometry, 8–13 Dec. 03

(This list consists of participants who attended computational part as well as the expository part)

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16. Shiva Datta	Govt. P.G. College, Nainital
17. Archana Morye	IIT, Mumbai

## Preface

This part of the volume is based on six lectures and tutorials that were prepared for a Workshop in Computational Commutative Algebra held at the Harish Chandra Research Institute (HRI), Allahabad, India in December 2003. The workshop was aimed at graduate students and was conducted in parallel to the Conference on Commutative Algebra and Combinatorics held at HRI from December 8–13, 2003. The material in the early chapters is based heavily on the research monograph *Gröbner Bases and Convex Polytopes* [Stu96] by Bernd Sturmfels. We have attempted to explain the key concepts in this monograph to students who are not familiar with either Gröbner bases or convex polytopes by building up the basics of these theories from scratch. The tutorials and examples are meant to help this development. There is a special emphasis on actual computations via various software packages.

Lectures 1, 3, and 5 were written and delivered by Rekha R. Thomas (University of Washington), and Lectures 2, 4, and 6 were written and delivered by Diane Maclagan (Rutgers University). The tutorials for the lectures were prepared and conducted by Tony Puthenpurakal of IIT Bombay and A.V. Jayanthan of HRI (Tutorial 1); Amit Khetan of the University of Massachusetts, Amherst (Tutorial 2); Leah Gold of Cleveland State University (Tutorials 3 and 5); and Sara Faridi of Dalhousie University (Tutorial 4). Amit Khetan also contributed to Lecture 6.

We thank the host institutions Bhaskaracharya Pratishthana and Harish-Chandra Research Institute, India for the invitation to conduct this workshop and their very warm hospitality. We also thank the students at the workshop for helpful comments and corrections. Special thanks to Amitava Bhattacharya, Tristram Bogart, Anders Jensen and Edwin O'Shea who helped to proofread the final version. Finally, we acknowledge Sara Faridi for editing and creating an index for the book, and Leah Gold for stellar proofreading.

DIANE MACLAGAN (Rutgers University)  
REKHA R. THOMAS (University of Washington).





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# Part II

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## TOPICS IN COMMUTATIVE ALGEBRA AND COMBINATORICS

---

*based on*

**International Conference on  
Commutative Algebra and Combinatorics**

*organised by*

**Bhaskaracharya Pratishthana, Pune  
Harish-Chandra Research Institute, Allahabad**

Lecture Notes

In Memory of *Vishwa Chander Dumir* (1943–2006)

**Winfried Bruns  
Editor**



# Preface

On the occasion of the first International Meeting of the American Mathematical Society in India held by the Indian Academy of Sciences and Indian National Science Academy at Indian Institute of Sciences, Bangalore from December 17–21 the Bhaskaracharya Pratishthana, Pune and the Harish-Chandra Research Institute, Allahabad organised an International Conference on Commutative Algebra and Combinatorics at Allahabad, 08–13 December, 2003.

This volume contains the notes of the four series of lectures given by W. Bruns, S. Hoşten, I. Swanson and N. V. Trung at the conference. T. Hibi was unable to attend the conference, but has kindly supplied notes for this volume in cooperation with H. Ohsugi.

The conference was a tribute to Vishwa Chander Dumir on the occasion of his 60th birthday. His work with H. Anand and H. Gupta on a combinatorial enumeration problem, called *magic squares* by R. Stanley, was one of the starting points of combinatorial commutative algebra. The Anand–Dumir–Gupta conjectures were proved by Stanley using methods of commutative algebra. This development led to a renaissance of the theory of Hilbert functions and made combinatorially defined algebras a focal point of commutative algebra.

The article by Bruns gives an introduction to the Anand–Dumir–Gupta conjectures and their proof. The notes by Hoşten surveys a large class of combinatorially defined ideals in polynomial rings, and the contribution by Ohsugi and Hibi is devoted to algebraic statistics, the perhaps most recent facet of combinatorial commutative algebra. Swanson treats primary decomposition, a classical subject going back to Lasker and Noether, for which computational methods have now become available. Castelnuovo–Mumford regularity, surveyed by Trung, is a very useful measure for the complexity of ideals and modules and of great interest in recent research.

While these notes were prepared, the sad news of Dumir’s untimely death on February 3, 2006 reached us. We are grateful to Professor R. J. Hans-Gill for writing an obituary. The great mathematician Dumir and his excellent work will be kept in lively remembrance by all of us.

The conference was partially supported by the following institutions:

National Board for Higher Mathematics, India  
Department of Science and Technology, Government of India  
International Mathematical Union  
National Science Foundation, U.S.A.

We are grateful to them for their generous support.

The editor and contributors of this volume are very grateful to the Bhaskaracharya Pratishthana for organizing an excellent conference, Our warm thanks go to Professor Shashikant A. Katre and Ms. Manjusha Joshi for their excellent handling of all the technical and organizational work that arises at such a large event.

We thank the Harish-Chandra Research Institute for providing a very pleasant and comfortable environment. We are deeply indebted to Professor Ravi S. Kulkarni, former Director of the Harish-Chandra Research Institute, for his encouragement during all the stages of organization and also for taking keen interest in publishing these notes in the lecture note series of the Ramanujan Mathematical Society.

Last, but not least, the contributors and the editor cordially thank Professor Jugal K. Verma. His constant efforts were a decisive factor for the success of the conference and the publication of these notes. He has been a driving force in promoting Commutative Algebra in the Indian mathematical community.

**March 2006**

**Winfried Bruns**

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