The Riemann zeta function and related themes

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Volume 2

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Proceedings of the international conference held at the National Institute of Advanced Studies, Bangalore, December 2003

Volume editors

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K. Srinivas

International Press
www.intlpress.com

Ramanujan Mathematical Society
Lecture Notes Series, Volume 2
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Published in 2010 by International Press, Somerville, Massachusetts, U.S.A. under license from the Ramanujan Mathematical Society.

This work is also published in the Republic of India, exclusively by the Ramanujan Mathematical Society.

ISBN 978-1-57146-187-2

Typeset using the LaTeX system.
Printed in the United States of America.

Foreword

As I have said on other occasions, in my opinion, K. Ramachandra is the real successor of Ramanujan in contemporary Indian mathematics. Following the tradition of Chowla, Pillai and Vijayaraghavan, he and his students have continued to make significant contributions to classical number theory. It is not surprising that active mathematicians from various countries came to attend the conference held at Bangalore in December 2003, to offer their felicitations to him on completing seventy years of meaningful life. I did not attend the conference, because I have been out of active mathematics for some time. It is, therefore, all the more gracious of the editors, R. Balasubramanian and K. Srinivas to give me the honour of writing this foreword to the conference proceedings. I not only admire Ramachandra for his mathematics, but also greatly value his friendship, and the regard and affection he has shown to me over the years.

Ramachandra, born in 1933, completed his B.Sc (Hons.) and M.Sc. studies at Bangalore. He got his Ph.D. in 1965 at TIFR under the supervision of K.G. Ramanathan, whose influence on Indian mathematics has been remarkable.

Ramachandra's initial work was on algebraic number theory. The reviewer M. Eichler, of his first paper: Some applications of Kronecker's limit formulas, Ann. of Math (2), 80 (1964), 104-148, started the review with the remark: "This paper contains some remarkable new results on the construction of the ray class field of an imaginary quadratic number field."

When the seminal work of Alan Baker appeared in the 1960's, he and his students, especially T.N. Shorey, took up transcendental number theory and made remarkable contributions to the Baker theory and its applications to problems of classical number theory.

Another important result proved by Ramachandra states: for m large, between m^2 and $(m+1)^2$ there is an n and a prime p dividing n such the $p > n^{1/2+1/11}$.

After 1974, he turned his attention to hard core classical analytical number theory, especially Reimann zeta function and general Dirichlet series. His school has made significant contributions, especially to gaps between ordinates of critical zeros of the zeta function and mean values of the fractional powers of the Riemann zeta function inside the critical strip. The method he developed to give a simple proof for the asymptotic formula for the fourth power mean and his application of a contour suggested by Huxley and Hooley has found wide applications.

With his contributions, it was natural for him to receive various honours like election to fellowships of various science academics, medals and award lectures, especially Ramanujan medals of Indian National Science Academy and Indian Science Congress, invitation to conferences and academic institutions. But to Ramachandra the excitement of doing mathematics is its own reward. I wish him many years of happiness of creative pursuit.

I am very grateful to Prof. K. Srinivas for his help in preparing this foreword.

– R.P. Bambah

PREFACE

This volume represents the proceedings of the international conference *Analytic Number Theory* held at National Institute for Advanced Studies, Bangalore, during December 13 - 15, 2003. On this occasion Professor K. Ramachandra, who turned 70, was felicitated. We thank TIFR (Mumbai) and HRI (Allahabad) who have co-sponsored this conference. We thank the participants of the conference and the contributors to this volume and the referees. Thanks are also due to the administrative staff at NIAS for their help in organising the conference. Our special thanks are due to Mr. Ramakrishna Manja, Ms. M. Geetha, Ms. R. Indra and Dr. R. Thangadurai who took upon themselves various organisational matters and followed them up meticulously. We thank Prof. S.D. Adhikari for some editorial help. Finally we would like to thank Ramanujan Mathematical Society to consider our proceedings to publish under the DST-sponsored RMS Lecture Notes series in Mathematics. Publication of the present volume is co-sponsored by The Institute of Mathematical Sciences, Chennai.

August, 2006

R. Balasubramanian
K. Srinivas

Felicitation Messages

Let me congratulate you on 70 years of fascinating mathematics. As soon as I entered research, 30 years ago, yours became a familiar name; and your influence has remained with me ever since.
Time permits me to mention in detail only one strand of your work - but it is one that clearly demonstrates how important your research has been. A little over 20 years back you proved the first results on fractional moments of the Riemann Zeta-function. At first I could not believe they were correct!! Since then however the ideas have been extended in a number of ways. They have lead of course to a range of important new results about the Zeta-function and other Dirichlet series. But just as significantly the ideas have led to new conjectures on the moments of the Riemann Zeta-function. These conjectures provide the first successful test for the application of Random Matrix Theory in this area. Nowadays this is a growth area which has contributed much to our understanding of zeta-functions. And it can all be traced back to your work in the late 1970's.
So, let me congratulate you once again, and send you every good wish for the future.

— Roger Heath-Brown

I have great respect and affection for Professor Ramachandra who is the father of analytic number theory in the second half of the twentieth century in India.

— Alladi Krishnaswamy

At the celebration of your 70th birthday I congratulate you on your many excellent results in number theory, algebraic, transcendental and, above all, analytic. You have served the mathematical community not only by your research and teaching, but also by founding and editing for many years, together with your eminent student, R. Balasubramanian, The Hardy Ramanujan Journal, one of very few privately published mathematical journals. Let me wish you many more fruitful years.

— Andrzej Schinzel

Your colleagues from Department of Mathematics and Mechanics of Lomonosov Moscow State University warmly and heartly wish to express sincere congratulations on occasion of your seventieth birthday.
We know you as the great number-theoretist, the friend of mathematicians from Russia, and the nice person. Your mathematical school has the great influence on the development of the Number Theory in the whole world. In the name of our friendship, please, accept our sincere appreciation and gratitude.

— G.I. Arkhipov, V.N. Chubarikov, A.A. Karatsuba:
Y.V. Nesterenko, A.B. Shidlovski

It is a great pleasure to congratulate you on the occasion of your 70th birthday.
You have brought in the large contribution to development of number theory our favorite science. Your results are widely known to experts on number theory in many countries. Scientific contacts to you always have the large interest for us.
We wish you kind health, pleasures in life, successes in all your projects and new creative achievements.

— The chair of number theory of Moscow state university:
A.B. Shidlovskii, Yu. Nesterenko, A. Galochkin,
N. Moshevitin, V. Zudilin

I was very pleased to hear about the conference in honor of Professor Ramachandra on the occasion of his 70th birthday. I have great respect for Professor Ramachandra and have long admired his work. I think it is splendid that he should be honored in this way.

— Alan Baker

The post-Ramanujan period in India saw great advances, in several areas in modern mathematics. In analytic number theory, a beginning was made by my teacher Professor Anada Rau of Chennai. He was a student of G. H. Hardy and a friend and contemporary of Ramanujan at Cambridge. Others who followed him include T. Vijayaraghavan, S. S. Pillai and his student Sathe (both of whom had their career ended by early death) and V. Ganapathi Iyer — to name a few.
In the contemporary period in India, Professor K. Ramachandra occupies a special place in the field of analytic number theory. Besides doing outstanding research work on the many difficult problems involving the Riemann zeta function, he along with his talented students and coworkers built an enduring research group of analytic theorists in India which already enjoys a high reputation. I heard it said in some quarters that this group is one of the potential centers from where to expect a proof or disproof of the Riemann conjecture. Ignoring temptations of moving to greener pastures outside his country, Ramachandra leads – like the wise men of yore – a life of simple living and high thinking. Like the famed late Sarvadaman Chowla, Ramachandra is full of modesty and humility. While self-importance reigns almost everywhere, he has no hesitation in declaring one of his students (Balu) as a greater mathematician than himself.
Let us all join in wishing Professor Ramachandra a long, healthy and mathematically fruitful life.

— M.V. Subbarao

I owe very much to you mathematically and personally, and in particular I gratefully remember my visit, through your initiative, to TIFR in 1985 which gave me a unique opportunity to mathematical work in a very pleasant and inspiring environment. Though I am afraid that because of other commitments I will be unable to participate your birthday conference, in any case I will be present in spirit and wish best success both to the conference and for you personally.

— Matti Jutila

I have been lucky to be associated with Professor Ramachandra starting from my student days and I wish him a very very long and happy life on this nice day.

— A. Sankaranarayanan

Doctoral Students of Professor K. Ramachandra

1. T.N. Shorey
2. S. Srinivasan
3. R. Balasubramanian
4. M. J. Narlikar
5. V.V. Rane
6. A. Sankaranarayanan
7. K. Srinivas

Table of contents