

Number theory

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Number theory

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Volume editors

S. D. Adhikari
R. Balasubramanian
K. Srinivas



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Preface

This volume represents the proceedings of the international conference *Analytic Number Theory with special emphasis on L-functions* held at The Institute of Mathematical Sciences, Chennai, in January 2002. Some invited speakers who could not accept our invitation have kindly contributed to this volume. On this occasion Professor M. V. Subba Rao, who turned 80, was felicitated. We thank the participants of the conference and the contributors to this volume and the referees. Thanks are also due to the administrative staff at the Institute of Mathematical Sciences for their help in organising the conference. Our special thanks are due to Dr. R. Thangadurai and Ms. R. Indra who took upon themselves various organisational matters and followed them up meticulously. Finally we would like to thank Ramanujam Mathematical Society to consider our proceedings as the first volume of the DST-sponsored RMS Lecture Series in Mathematics. Publication of the present volume is co-sponsored by The Institute of Mathematical Sciences, Chennai.

January 2005

**S. D. Adhikari
R. Balasubramanian
K. Srinivas**

Foreword

I thank Professor Balasubramanian for inviting me to write a foreword to these proceedings of a number theory conference, held at the Institute of Mathematical Sciences, Chennai, during which Professor M. V. Subbarao, who turned eighty, was felicitated.

Professor Subbarao was born on May 4, 1921 in the village Yazali near Bapatla (Guntur District), India. His father was Shri Mathukumalli Narasimha Rao and his mother Venkata Subbamma. He completed his high school education at Guntur and a Master's degree at Presidency College, Madras in 1941, and subsequently entered the department of mathematics of University of Madras to work with Professor Vaidyanathaswami for his doctoral degree.

Later, after working for some years at Presidency College, Madras, he joined the Department of Mathematics, Sri Venkateshwara University, Tirupathi, where he served as professor before moving to the University of Missouri, Columbia (U.S.A.) and later to University of Alberta, Edmonton, Canada in 1963. He spent one year on invitation as Professor and Department Head at Kerala University (1966–67). Professor Subbarao and his wife Mrs. Suseela, have a son, Professor M. Vidyasagar, and a daughter Savithri.

Dr. Subbarao's doctoral thesis is on Functional Analysis and in the beginning he wrote a few papers in Analysis and Topology, however, for most of his mathematical research career he has worked in Number Theory. In particular, Professor Subbarao has contributed many important results and conjectures to the theory of arithmetical functions. He has several works dealing with the distribution of generalized K -free integers, partition functions, the Scholz-Brauer problem in addition chains. On the partition function $p(n)$, he had joint papers with George Andrews, D. Bressoud, B. Richmond, L. Carlitz, E. Straus, J. Fabrykowski, A. K. Agarwal et. al., in addition to some forthcoming papers jointly with Imre Katai. In this context Professor Subbarao made the following important conjecture.

For every positive integer m , on every arithmetic progression $r \pmod{m}$; $0 \leq r < m - 1$, $p(n)$ assumes both even and odd values infinitely often.

For the cases $m = 2, 3, 4, 5, 6, 8, 10, 12, 16, 20, 40$, the conjecture had been established by the efforts of many mathematicians. In fact, the works of Subbarao, Hirschhorn and Subbarao and Hirschhorn among others like Frank Garvan and D. Stanton established these cases by elegant combinatorial methods.

This conjecture generated a lot of research work. We mention the following result of K. Ono, which is a major achievement in this direction.

For every positive integer m , in every arithmetic progression $r \pmod{m}$, $0 \leq r < m - 1$, $p(n)$ assumes even values infinitely often. Also, if $p(n)$ assumes odd value for a single n in an arithmetical progression, then $p(n)$ assumes odd values for infinitely many n in that arithmetical progression. Quantitative versions of the above result were obtained later by J. P. Serre and S. Ahlgren. The odd case of the conjecture is still open, but has been verified for all $m \leq 10^5$. Professor Subbarao offers a \$500 for a complete proof of his conjecture. He has an analogous conjecture for product partitions.

Many other conjectures and unsolved problems appear in his papers with Erdős, Straus, Katai, Hardy et. al. We just mention one more of his conjectures. If p_1, \dots, p_r are any distinct primes and a_1, \dots, a_r positive integers, then $\prod(p_i^{a_i} - 1)$ divides $((\prod p_i^{a_i}) - 1)$ only if $r = 1$.

Professor Subbarao collaborated with some forty other researchers, including Paul Erdős, Ernst Straus, L. Carlitz, George Andrews, Imre Katai, Wang Yuan, George Hardy, David Brossoud, M. D. Hirschhorn, K. G. Ramanathan, K. Ramachandra, R. Balasubramanian, R. A. Smith, H. L. Abbott, D. Suryanarayana, R. Sitaramachandra Rao, V. Sivarama Prasad, V. Sitaramaiah, A. Verma, A. K. Agarwal, V. V. S. Sastri, M. Sugunamma et. al. besides his son Vidyasagar. His research publications total around 200.

His work and his pleasant nature brought Professor Subba Rao a number of collaborators. In particular, Professor Subba Rao's Erdős number is one. Indeed, he has eight joint papers with Professor P. Erdős. A number of Indian mathematicians, among them Professor K. G. Ramanathan, Professor R. Balasubramanian and myself, have had the good fortune of working with Professor Subbarao. He has had special interest in Mathematics departments of Sri Venkateshwara and Andhra Universities where his support and encouragement produced excellent mathematicians such as Professors D. Suryanarayana and R. Sita Ramachandra Rao.

K. Ramachandra

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