Elementary Number Theory

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by Jie Xiao (Memorial University of Newfoundland)

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Preface

Traditionally, elementary number theory is a branch of number theory dealing with the integers without use of techniques from other mathematical fields. With this objective in mind, and exercising as much control as possible over my own prejudices, I have sought to pare away all material that might be considered extraneous in a three hour per week, twelve week semester course in elementary number theory. This leads to my aim in writing this book: On the one hand, I must present in a well-motivated and natural sequence the basic ideas and results of elementary number theory. On the other hand, enough material is covered to provide a firm base on which to build for later studies in algebraic number theory and analytic number theory. The only background material required of the reader is a knowledge of some simple properties of the system of integers. Otherwise this concise book is self-contained.

The book begins with a few preliminaries on induction principles, followed by a quick review of division algorithm. The substance of the book starts in the second chapter, where, using divisors, the greatest (least) common divisor (multiple), the Euclidean algorithm and linear indeterminate equation are discussed. This foundation supports the subsequent chapters: prime numbers; congruences; congruent equations; and, finally, three additional topics (comprising cryptography, Diophantine equations and Gaussian integers). Placed at the end of each chapter are some exercises that illustrate the theory and provide practice in the techniques. Answers to all the even-numbered problems are given at the end of the book.

The above-mentioned material was used with groups of undergraduates in one-semester courses at Memorial University of Newfoundland. A brisk pace should make it possible to cover this little book in its entirety in one semester.

While writing this book I was encouraged by and benefited from a number of individuals. Here, I want to thank Hershy Kisilevsky at Concordia University as well as Herb Gaskill, Donald E. Rideout, Yiqiang Zhou and my students at Memorial University of Newfoundland for their helpful comments and suggestions. Meanwhile, I am very grateful to the referee for his invaluable review, but also to Shing-Tung Yau at Harvard University and Brian Bianchini and Lisa Lin at International Press for their practical advice. Last but not least, I wish to take this opportunity to acknowledge

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