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Volume IX

Surveys in Differential Geometry

Eigenvalues of Laplacians and other geometric operators

edited by
Alexander Grigor'yan and Shing-Tung Yau

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Preface

This year, we have decided to concentrate on an important topic in Geometry: the eigenvalues of the Laplacian and other geometric operators. This has been one of the most fundamental subjects in the area. The works on Hodge theory and the heat equation proof of the Hodge theorem certainly show its importance. One of the most dramatic and influential papers was the one by Mark Kac on "How to hear the shape of the drum", which intimately connected the geometry of a manifold with the spectrum of the Laplacian on it.

A strong understanding of spectral properties of differential operators has deep consequences in Geometry, Physics, Number Theory, Probability Theory, etc. Analytic methods have penetrated also into Discrete Mathematics, where the study of the spectral properties of difference operators leads to deeper understandings of the combinatorial questions on graphs.

This volume is an attempt to survey the diversity of directions unified under the above headings. Although the subject is too big to be covered by a single book, we still believe that the present collection does introduce a substantial part of the subject, and we hope it will be beneficial to geometers and experts from the related areas.

Alexander Grigor'yan and Shing-Tung Yau
April 2004

Contents

Preface iii

Anomalous diffusion and stability of Harnack inequalities
Martin T. Barlow 1

From isoperimetric inequalities to heat kernels via symmetrisation
Gérard Besson 27

Discrete Isoperimetric Inequalities
Fan Chung 53

An excursion into geometric analysis
Tobias H. Colding and William P. Minicozzi II 83

Eigenvalues of elliptic operators and geometric applications
Alexander Grigor'yan, Yuri Netrusov, and Shing-Tung Yau 147

Spectral gap, logarithmic Sobolev constant, and geometric bounds
Michel Ledoux 219

Discrete Analytic Functions: An Exposition
László Lovász 241

Conformal properties in classical minimal surface theory
William H. Meeks III and Joaquín Pérez 275

Analysis of the Cut Locus via the Heat Kernel
Robert Neel and Daniel Stroock 337

Analysis on Riemannian co-compact covers
Laurent Saloff-Coste 351

Functoriality and Small Eigenvalues of Laplacian on Riemann Surfaces
Freydoon Shahidi 385

The inverse spectral problem
Steve Zelditch 401