

## Surveys in Differential Geometry

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Volume VII

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*Surveys in*  
**Differential Geometry**

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Papers dedicated to Atiyah, Bott,  
Hirzebruch, and Singer

edited by  
Shing-Tung Yau

Surveys in Differential Geometry, Vol. 7

Editor:  
Shing-Tung Yau, Harvard University

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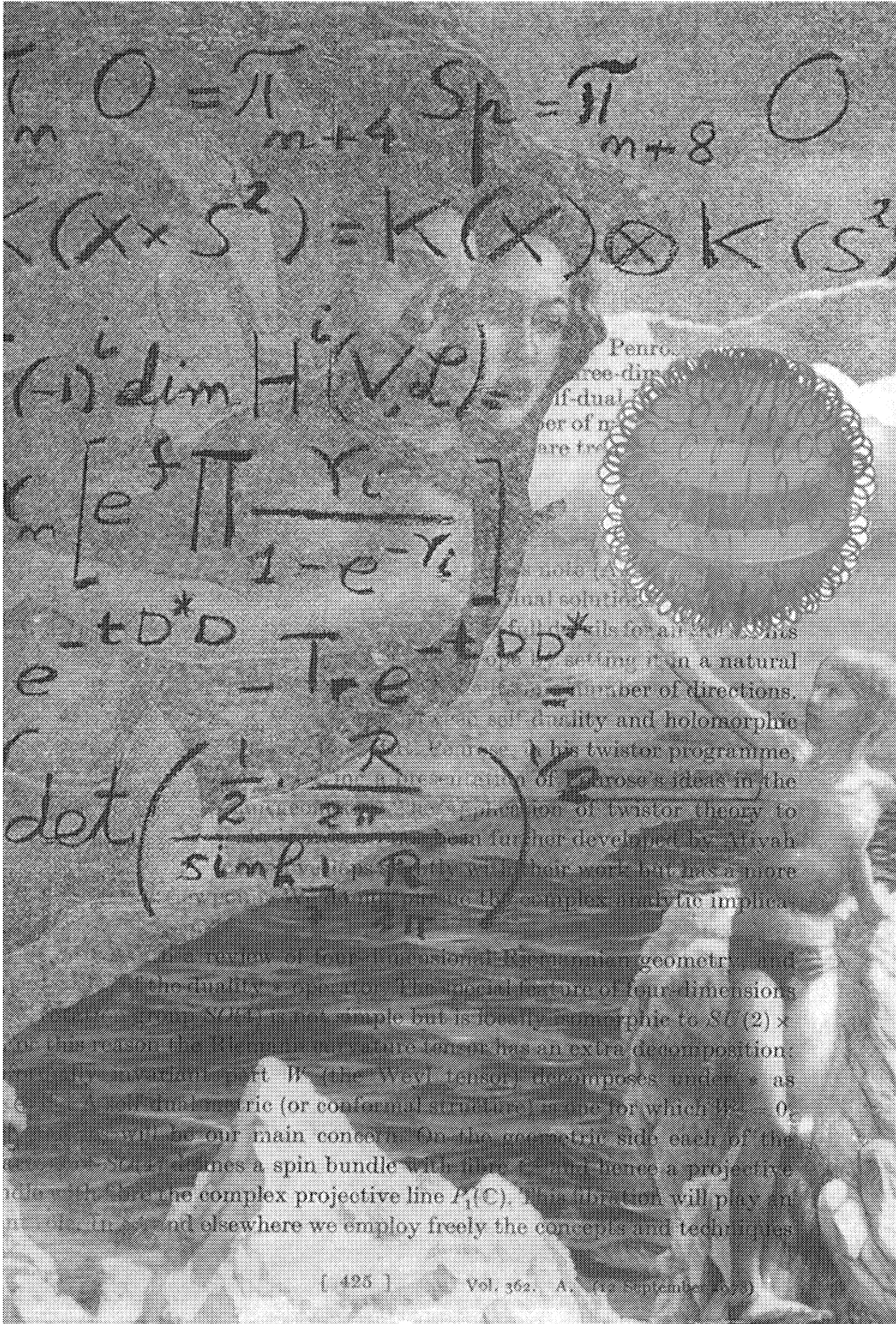
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ISBN 978-1-57146-178-0

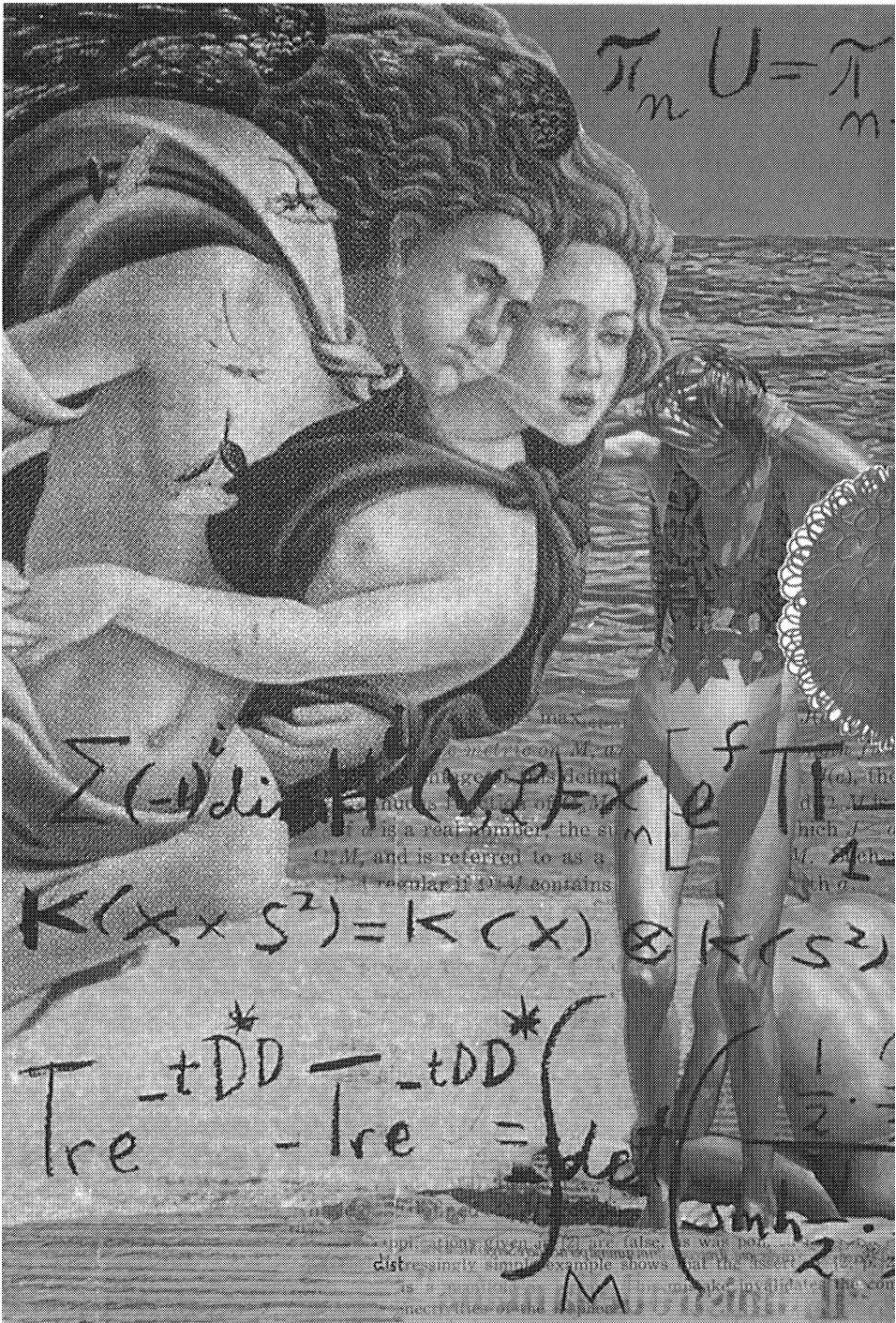
*Paperback reissue 2010. Previously published in 2000 under ISBN 1-57146-069-1 (clothbound).*

Typeset using the LaTeX system.





Art by Milen Poenaru  
 Dedicated to the index theory founders



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arithmetischen Geschlechtes verdankt  
 1937 zeigte, daß das arithmetische  
 Klassen von EGER-TODD [39]  
 nicht vollständig. Er beruht  
 in der Literatur kein voll-  
 definiert als eine Klasse von  
 von  $2n - 2$ ; bezüglich einer  
 "log" impliziert nicht  
 ist  $K(V_n)$  diese  
 Divisor bei  
 $n$ -Port  
 die bekannte  
 $2n$ -dimensionale  
 Klasse der Dimen  
 Neben Cohomologieklassen  $V_n$   
 "Eger-Toddsche  
 über  $\mathbb{C}$  in der  
 (CHERN [10], HODGE [20])  
 nur die CHERNSchen Klassen, und die  
 Klassen mit den CHERNSchen Klassen  
 ohne Belang. Das Toddsche Ge  
 der CHERNSchen Klassen definiert und  
 Arbeit, dann zu zeigen, daß  $\chi(V_n) = e(V_n)$ .  
 von  $T(V_n)$  erfolgt so: Man definiert in einer  
 Weis (S) ein bestimmtes Polynom in den CHERN  
 Klassen  $c_i$  vom Gewicht  $n$  im  $\mathbb{C}$ -Rationalen  
 Kre im Sinne des Cohomologieringes von  $V_n$ . Dieses Polynom  
 $n$ -dimensionale Cohomologieklasse von  $V_n$ , deren Wert auf  
 $2n$ -dimensionalen Fundamentalzyklus<sup>\*)</sup> von  $V_n$  per Definitionem  
 gleich  $\chi(V_n)$  ist. Nach Definition  $\chi(V_n) = \sum_{i=0}^{2n} (-1)^i \dim H^i(V_n; \mathbb{C})$   
 Es ist eine nicht-triviale Tatsache, daß  $T(V_n)$  (für algebraische Mannig-  
 faltigkeiten  $V_n$ ) immer eine ganze Zahl hat. Man hat  
 $T(V_1) = \frac{1}{2} c_1[V_1]$ ,  $T(V_2) = \frac{1}{12} (c_1^2 + c_2[V_2])$ ,  $T(V_3) = \frac{1}{24} (c_1^3 + c_1 c_2[V_3] + \dots)$   
 \*) Jede komplexe Mannigfaltigkeit  $V_n$  ist in natürlicher Weise orientiert  
 mit  $x_1, \dots, x_n$  mit  $z_k = x_{2k-1} + i x_{2k}$  lokale komplexe Koordinaten  
 natürliche Orientierung durch die Reihenfolge  $x_1, x_2, \dots, x_{2n}$  geben  
 oder in anderen Worten durch  $dx_1 \wedge dx_2 \wedge \dots \wedge dx_{2n}$  ( $\mathbb{R}$ -orientatives  
 element). Wir verwenden im folgenden  $\int_{V_n}$  für die natürliche Orientierung. Erst  
 nach Vorzeichen für eine Orientierung ist die einer Homologieklasse, entsprechen  
 Cohomologieklasse eindeutig mit Vorzeichen festgelegt.  
 \*) Durch die natürliche Orientierung wird ein Element  $\alpha$  der  $2n$ -dimensionalen  
 ganzzahligen Homologiegruppe  $H_{2n}(V_n; \mathbb{Z})$  ausgezeichnet ( $2n$ -dimensionaler Funda-  
 mentalzyklus). Der Wert einer  $2n$ -dimensionalen Cohomologieklasse  $a$  auf dem  
 Fundamentalzyklus wird mit  $\langle a, \alpha \rangle$  bezeichnet.

$$\sum (1) \det \frac{\partial (y_i, z_j)}{\partial (x_i, v_i)}$$

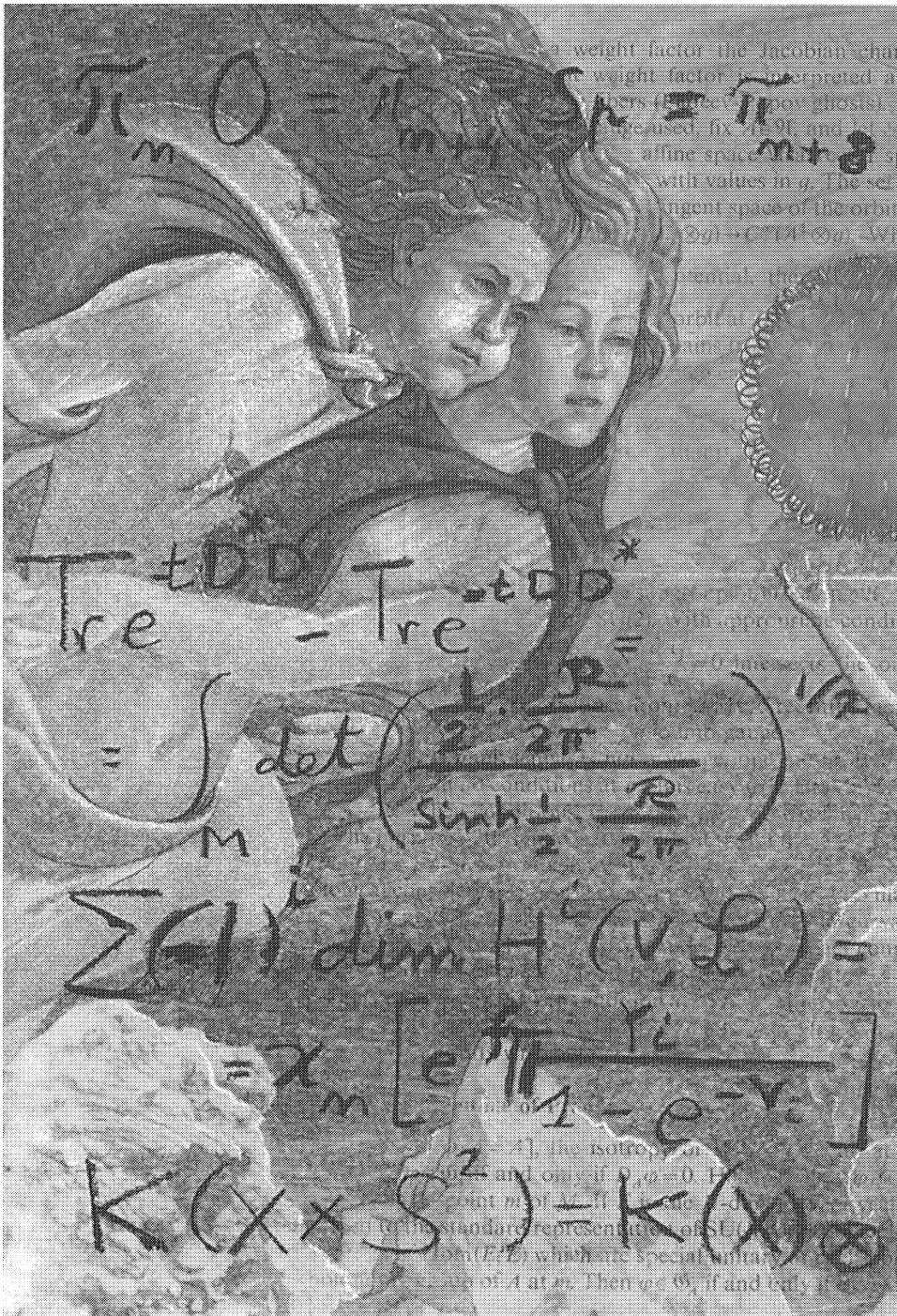
$$K(S \times X) = K(S) \otimes K(X)$$

$$\int_{V_n} \dots = \int_{m+1}^n \dots = \int_{m+1}^n \dots = 0$$

$$\left( \frac{\det \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 2 & \dots & 2\pi \end{pmatrix}}{2 \cdot 2\pi} \right)^{1/2}$$

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## **Preface**

This volume arises out of the conference sponsored by the Journal of Differential Geometry and held at Harvard University to honor the four mathematicians who founded Index Theory. A large number of geometers gathered for this historic occasion which included numerous tributes and reminiscences which will be published in a separate volume. The four men who together created Index Theory: Michael Atiyah, Raoul Bott, Frederich Hirzebruch, and Isadore Singer, were sources of inspiration, mentors and teachers for the other speakers and participants at the conference. The larger than usual size of this volume derives directly from the tremendous respect and admiration felt for the honorees.

Along with this volume, we give our best wishes to each of the honorees that they might have many more years to continue their own research and to inspire and encourage future mathematicians.

Sincerely,

S.-T. Yau



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