Advanced Lectures in Mathematics Volume XI

Recent Advances in Geometric Analysis

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ISBN 978-1-57146-143-8

Typeset using the LaTeX system. Printed in the USA on acid-free paper.

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Preface

"2007 International Conference in Geometric Analysis" was held in Taiwan University from June 18th to 22nd, 2007. This conference is sponsored by Mathematics Division, Center for Theoretical Sciences (NCTS) Taipei Office, Taida Institute for Mathematical Sciences (TIMS), Academia Sinica, Central University and Tsing-Hua University (Xinzu).

Geometric analysis studies functions, maps, tensors, and submanifolds governed by natural differential equations. A good understanding of these objects reveals important information of analytical and geometric structures, and has many implications in physics, algebraic geometry and topology.

In recent years, we have witnessed a great success of geometric analysis, the most important event being the solution of the Poincare conjecture by the Ricci flow. This shows the power of geometric partial differential equations in resolving some deepest problems in topology.

The aim of "2007 International Conference in Geometric Analysis" is to gather leading experts to discuss and exchange new progress and ideas on various topics in the field. This proceeding is an account on recent advances in geometric analysis and related equations, including Ricci flow, affine normal flow, geometric analysis on pseudoconvex hypersurfaces, Alexandrov space, manifolds with special holonomy, and singular plateau problem.

We would like to take this opportunity to thank Prof. Shing-Tung Yau for the support of publishing this proceeding. We also want to thanks all the authors for contributing this proceeding.

> Yng-Ing Lee Chang-Shou Lin Mao-Pei Tsui April 2009

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