

Advanced Lectures in Mathematics  
Volume X

# Trends in Partial Differential Equations

Editors: Baojun Bian, Shenghong Li, and Xu-Jia Wang

 International Press  
[www.intlpress.com](http://www.intlpress.com)



高等教育出版社  
HIGHER EDUCATION PRESS

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ISBN 978-1-57146-142-1

Typeset using the LaTeX system.  
Printed in the USA on acid-free paper.

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# Preface

This volume of 15 contributed papers is a tribute to Professor Guangchang Dong on the occasion of his 80th birthday. In the course of his almost 60 year career in mathematical research, his influence on the development of partial differential equations in China has been immersed, at both the teaching and the research level. To celebrate his 80th birthday, an international conference “Elliptic and Parabolic Equations and Applications” was held at the Center for Mathematical Sciences, Zhejiang University, Hangzhou, in August 2008. Some papers in this volume are contributed by speakers of the conference, and others by his friends and former students.

Professor Dong was born in January 1927 and received his Bachelor degree in 1950 at the Department of Mathematics, Zhejiang University, where he has been a faculty member since then. In 1952 he published his first two papers in analytic number theory in *Acta Mathematica Sinica*, a top mathematical journal in China. He kept working in number theory and by 1956 he had published a series of papers, see the list of his publications at the end of this book.

We have to mention an event which interrupted his research by a great deal. The Mathematics Department of Zhejiang University was a top one in China in early 1950s. But in 1952 the Chinese government decided to move all its faculty members, and also the library books, to Fudan University, Shanghai, except a small group of people remaining for engineering mathematics teaching. Prof. Dong was one of them. Considering the inconvenience in communication those days, one can imagine how difficult it was for a person at his twenties to work alone in mathematics. But his enthusiasm and talent in mathematics got him rewarded; his work in number theory was recognized and highly praised by Hua Loo-Keng, the leader of Chinese mathematics, which was undoubtedly a great impetus for him to continue his research.

However his research direction changed in the middle of 1950s. In 1954~1955 there was a program in partial differential equations (PDEs) at Beijing and Prof. Dong was sent to study PDEs. He was doing so well in the program that soon after he was able to write papers in the new area, and two were published again in *Acta Mathematica Sinica* in 1956. He then worked in the area for nearly 10 years, on different topics but mainly in degenerate elliptic and hyperbolic equations, and

equations of mixed type.

Unfortunately his research was suspended in early 1960s. From early 1960s and during the Cultural Revolution, for more than 15 years, he was assigned to study practical mathematical problems arising in industry, and for many days he had to live in factories. One of the projects he was working on was to establish and implement smooth approximations of the shape of ships by spline functions; another one was computer aided design. He had no publications in this period but some of his work is described in book [1].

The Cultural Revolution ended in late 1976, and he was able to return to mathematics at his fifties. It was since then he was able to focus on his research. In the early 1980s, he got the precious opportunity to visit USA for two years, mainly at the Courant Institute. Since then his interest changed from degenerate elliptic and hyperbolic equations to elliptic and parabolic equations, and he has done work in variational problems, subsonic flows, degenerate parabolic equations, viscosity solutions of fully nonlinear elliptic equations. Since the 1990's he also found interests in applications of PDEs in image processing.

A list of his publications is included at the end of the book, among which there are 4 textbooks or research monographs, and more than 60 papers, most of them were published after the 1980s. Looking back to the political era of the 1950s and 1960s, it is hard to imagine the difficulty for a person to concentrate on research, in particular for a person like Dong who had only a Bachelor degree and worked in an isolated environment. All his achievements should be contributed to his talent and his passion in mathematics and his unremitting effort. After nearly 10 years of retirement, he can still be seen everyday in his office in the Institute of Mathematics, Zhejiang University. His persistent effort has also brought him numerous honors and awards. In 1981 he was among the first group of academic people who received the eligibility from the Chinese government to supervise PhD students. Also mainly due to him, Zhejiang University was one of the few top institutions that obtained the eligibility to host mathematical postdoctoral fellows.

He has not only made great achievement in research. He also made Zhejiang University a center of partial differential equations in China and attracted many young people to Hangzhou to study or work with him. He was the principal proponent of the creation of the Mathematical Institute of Zhejiang University. He was a steering committee member of the Chinese Mathematical Society, and was a founding member for the Chinese SIAM (Society for Industry and Applied Math-

ematics). He has also served as the first Editor-in-Chief of the journal: *Applied Mathematics*, a Journal of Chinese Universities, and in the editorial board for many other journals, including the *Chinese Annals of Mathematics* and *Journal of Partial Differential Equations*. Currently he is the honorary president of the Mathematical Society of Zhejiang Province.

There are 15 papers in the volume; most of them are survey or expository papers. We hope these papers can help readers to catch up recent developments in related topics in elliptic and parabolic partial differential equations and their applications. We would like to take this opportunity to thank the Center for Mathematical Sciences for sponsorship of the international conference for Professor Dong's 80th birthday, and thank all participants of the conference. In particular we wish to thank all the authors for their efforts to make this book possible.

Baojun Bian, Shenghong Li, Xu-Jia Wang (editors)

March 31, 2009





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