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Advanced Lectures in Mathematics
Volume IX

Automorphic Forms and the Langlands Program

edited by

Lizhen Ji, Kefeng Liu, Shing-Tung Yau, and Zhu-Jun Zheng

International Press
www.intlpress.com

高 等 教 育 出 版 社
HIGHER EDUCATION PRESS

Advanced Lectures in Mathematics, Volume IX
Automorphic Forms and the Langlands Program

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ISBN: 978-1-57146-141-4

Printed in the United States of America.

15 14 13 12 2 3 4 5 6 7 8 9

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Preface

Classical modular forms on the upper half plane with respect to the modular group $SL(2,\mathbb{Z})$ and its congruence subgroups have arisen naturally in number theory, complex analysis, topology, mathematical physics and many other subjects. A natural generalization to other reductive and semisimple linear algebraic groups and their arithmetic subgroups consists of automorphic forms.

The closely related automorphic representations are basic notions in the celebrated Langlands program, which was proposed by Langlands in the late 1960 and has since revolutionized the fields of number theory, arithmetic algebraic geometry and representation theory. Indeed, many new instances of functoriality have been proved recently, which have led to some spectacular applications.

A more recent development is the geometric Langlands program, which has had many unexpected applications in algebraic geometry and mathematical physics.

To help younger mathematicians and non-experts to learn these important fields and their applications, an international conference titled *Langlands and Geometric Langlands Program* was organized in Guangzhou in June 18-21, 2007.

The speakers of the conference consist of Roman Bezrukavnikov, Alexander Braverman, Sol Friedberg, Dennis Gaitsgory, Stephen Gelbart, Li Guo, Yi-Zhi Huang, Erez Lapid, Ivan Mirkovic, Siye Wu and Jing Yu.

The papers in this book deal with the original Langlands program and consist of expanded lecture notes and papers by some of the speakers and other experts. We expect that this volume will be a useful reference to many people who want to learn these important subjects.

We would like to thank all the contributors for their efforts in carefully preparing their papers and lecture notes and the referees for their kind help. We would also like to thank the journal *Pure and Applied Mathematics Quarterly* for its generous support for the conference, and its staff members Emma Fang and Yin Sun for their help during the conference and in the process of editing and typesetting this book.

Managing Editor, Lizhen Ji
Kefeng Liu
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November 2008

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