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In Memory of Gene H. Golub

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Preface

An international workshop on computational mathematics and finance took place at University of Macau from 15th to 16th of December 2006. It was organized by the Department of Mathematics, University of Macau for the 25th anniversary of the university. The main theme of this workshop is the latest development in *computational mathematics and applications* and the papers involved in this proceedings cover this theme. There are eight invited speakers, four from Hong Kong, three from mainland China and one from Russia. The following is the list of invited speakers:

Prof. Raymond H. Chan of Chinese University of Hong Kong, Hong Kong.
Prof. Zhi-ming Chen of China Academy of Science, Beijing, China.
Prof. Wen Li of South China Normal University, Guangzhou, China.
Prof. Michael K. Ng of Hong Kong Baptist University, Hong Kong.
Prof. Wei-wei Sun of City University of Hong Kong, Hong Kong.
Prof. Eugene E. Tyrtyshnikov of Russia Academy of Science, Moscow, Russia.
Prof. Yi-min Wei of Fudan University, Shanghai, China.
Prof. Qiang Zhang of City University of Hong Kong, Hong Kong.

There are 11 papers in this proceedings. The main contents of 6 papers, provided by invited speakers, are described briefly as follows.

A certain research paradigm suggests an approximate representation of initial and final data in numerically viable low-parametric formats to be maintained during computation. One of purposes is to show how theory and methods obtained recently for rank-structured matrices can be developed into theory and methods for rank-structured tensors. Specifically, **E. Tyrtyshnikov** exposed a theory behind the 3D cross approximation algorithm together with its application to compression of petabyte-size data (1 petabyte = 2^{50} bytes) and preliminary results of design of iterative methods with rank-structured vectors.

Y. X. Wang, K. Du and **W. W. Sun** investigated fast algorithms for electromagnetic scattering from a two-dimensional rectangular open cavity embedded in an infinite ground plane. By introducing a transparent (artificial) boundary condition (Dirichlet-to-Neumann map), they reduced the problem in the open cavity to a bounded domain problem. Fast algorithms based on FFT were designed for solving the resulting discrete systems of the cavity models with layered media. Numerical experiments showed that the method is efficient and easy to implement as a preconditioner for cavity models with more general media.

E. S. Fung and **M. K. Ng** studied an interactive hidden Markov Model (IHMM) which is a variant of HMM. In the model, the observable symbols are affected by the hidden states and the hidden states are also affected by the observable symbols. They proposed and employed Nonnegative Matrix Factorization (NMF) techniques to calibrate model parameters in IHMM. Numerical results were presented to illustrate NMF for IHMMs.

W. Li presented a computable formula of the structured backward errors for the discretization system of electromagnetic scattering from a large cavity. He showed that the structured stability of a method for solving this system is independent of the (1,1)-block structure of the coefficient matrix.

Inexact Krylov subspace methods are very effective in some scientific applications where the inexact matrix-vector product appears naturally. Several authors have studied different aspects of the use of inexact matrix-vector product in iterative methods. Using the Kronecker product-based technique, K. Du and **Y. M. Wei** generalized the existing results to global methods for matrix equations. The corresponding methods are called inexact global Krylov subspace methods. Some numerical experiments were reported to illustrate the results.

X. Q. Jin, W. Wang and **Y. M. Wei** analyzed the convergence rate of the GMRES method theoretically and numerically.

The success of the workshop should be credited to the enthusiastic participation of the attendees. The editors would like to thank all who contributed to this proceedings. We wish that the publication of the proceedings of the workshop may provide some inspiration and shed new light on current research in computational mathematics. The publication of the proceedings is supported by the research grant RG-UL/07-08S/Y1/JXQ/FST from University of Macau.

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