

Advanced Lectures in Mathematics  
Volume VI

# Geometry, Analysis and Topology of Discrete Groups

Editors: Lizhen Ji, Kefeng Liu, Lo Yang, Shing-Tung Yau



International Press  
[www.intlpress.com](http://www.intlpress.com)



高等教育出版社  
HIGHER EDUCATION PRESS

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*2000 Mathematics Subject Classification.* 00-02.

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ISBN 978-1-57146-126-1

Typeset using the LaTeX system.  
Printed in the USA on acid-free paper.

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# Preface

Discrete subgroups  $\Gamma$  of Lie groups  $G$  are foundational objects in modern mathematics and occur naturally in different subjects. For example, integers form a discrete subgroup  $\mathbb{Z}$  of the Lie group  $\mathbb{R}$ . They give rise to actions on homogeneous spaces associated with the Lie groups  $G$ . These actions of  $\Gamma$  can be used to study the discrete groups  $\Gamma$  and are also interesting on their own. For example, when  $G$  is a noncompact semisimple Lie group, there is an associated Riemannian symmetric space  $X = G/K$ , and the quotient  $\Gamma \backslash X$  is a locally symmetric space, which is special Riemannian manifolds and often occur as moduli spaces in algebraic geometry and number theory.

To understand better the structures and applications of discrete subgroups of Lie groups and locally symmetric spaces, an instructional conference titled *Geometry, Topology and Analysis of Locally Symmetric Spaces and Discrete Groups* was held from July 17 to August 4, 2006 at the Morningside Center of Mathematics in Beijing. The conference consisted of lecture series and research talks by experts around the world. This proceedings contains expanded lecture notes and related papers of some of the talks on the conference. These papers cover many topics related to the themes of this conference, such as algebra, analysis, geometry, number theory and topology. We would like to thank all the speakers for their stimulating talks and especially for the authors of the papers included in this book for their efforts in writing up their lecture notes. We hope that they will present a global perspective on the beautiful subjects around discrete subgroups of Lie groups and locally symmetric spaces.

This three-week long conference was generously funded by the Morningside Center of Mathematics, and also partially sponsored by the Higher Education Press in China and the journal *Pure and Applied Mathematics Quarterly*. We would like to thank the staff members of the center, in particular Xiaoning Li, and the secretaries of the journal, in particular Chen Fang, for their devoted help before and during the conference. Without them, this conference could not run smoothly.

Managing Editor, Lizhen Ji  
Kefeng Liu  
Lo Yang  
Shing-Tung Yau  
November 2007



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