Advanced Lectures in Mathematics Volume II

Asymptotic Theory in Probability and Statistics with Applications

Editors: Tze-Leung Lai, Lianfen Qian, Qi-Man Shao





Tze-Leung Lai Department of Statistics Stanford University Qi-Man Shao Department of Mathematics University of Oregon

Lianfen Qian Department of Mathematical Sciences Florida Atlantic University

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Preface

To celebrate the 65th birthday of Professor Zhengyan Lin, an International Conference on Asymptotic Theory in Probability and Statistics was held at the Center of Mathematical Sciences and Department of Mathematics of Zhejiang University, Hangzhou, China, in the summer of 2006. One of the aims of the conference was to provide a platform for the exchange of new ideas and recent developments in asymptotic theory and applications. Many speakers of the conference were invited to contribute to this volume, which consists of expository papers based on their invited talks or related research areas. All papers were carefully peer refereed. We would like to dedicate this book to Professor Zhengyan Lin and wish him continuing success in many years to come!

Professor Lin is a leading probabilist in China. He has made significant contributions to the development of asymptotic theory, especially, limit theorems for mixing dependent random variables and selfnormalized sums, and sample path properties of Gaussian processes. Professor Lin has published over 140 papers and 7 books. He and Professor Chuanrong Lu have supervised over 160 graduate students at Hangzhou University (now merged with Zhejiang University) since 1982.

An objective of the present volume of 18 papers by the invited speakers and contributors to the conference is to introduce graduate students to some active research areas in probability and statistics. Most papers are survey papers so that the present volume can provide readers with a valuable resource in probability, statistics and their applications. Obviously, we cannot cover all of the important topics of current research.

The volume consists of three parts: (I) Limit Theorems, (II) Statistics and Applications, and (III) Mathematical Finance and Insurance.

Part I has 8 papers, focusing on limit theory through various angles. It starts with the probability theory of self-normalization (Lai and Shao), followed by random partitions (Su), adaptive designs (Zhang), Gaussian processes (Wang), Gaussian random fields (Xiao), large deviations theory for two-parameter Gaussian processes (Chen and Csörgő), intersection local times (Chen), and ends with limit theorems for *U*-statistics (JING) to serve as a link between probability and statistics.

Part II contains 7 papers, exploring various research areas in statistics and its applications. It starts with the classical inverse problem for the *t*-Statistics (Yang, Fang and Kotz), followed by statistical issues in rounded data (Bai, Zheng, Zhang and Hu). Then it introduces a variety

Preface

of useful statistical models such as piecewise regression models (Qian), partially linear models (Wang, Liang and Jin) and nonlinear time series models (H. Yu). Finally it reviews recent work on classification and illustrates a probabilistic classifier with environmental and remote sensing applications (J. Yu and Ranneby), mixed linear model approaches for complex trait analysis (Yang and Zhu).

Part III consists of 3 papers. It begins with introducing inference and computation for stochastic volatility models related to option pricing (Ji), and then gives an overview of Choquet integrals and their applications to risk theory (Wang and Yan). The volume ends with a paper by Yang on actuarial science and its recent developments.

We thank all authors for their superb contributions and the referees for their thorough and timely work. Thanks also go to the Center of Mathematical Sciences at Zhejiang University for providing conference facilities and financial support, to the executive editors, S. T. Yau, K. F. Liu and L. Z. Ji, for their interests in publishing the volume for the Advanced Lecture in Mathematics, and to the Chinese National Science Foundation for financial support. Last but not the least, we thank the local organizers for their tremendous contributions to the conference.

> T. L. Lai, L. F. Qian and Q. M. Shao March 2007

List of contributors:

Zhidong Bai	Northeast Normal University, China
Ranneby Bo	Swedish University of Agricultural Sciences, Sweden
Bin Chen	Carleton University, Canada
Maklós Csörgő	Carleton University, Canada
Xia Chen	The University of Tennessee, USA
Kai-Tai Fang	Zhuhai Campus of Beijing Normal University, China
Guorong Hu	Northeast Normal University, China
Chuanshu Ji	University of North Carolina, USA
Lei Jin	Texas A&M University, USA
Bing-Yi JING	Hong Kong University of Science and Technology,
	Hong Kong, China
Samuel Kotz	George Washington University, USA
Tze Leung Lai	Stanford University, USA
Hua Liang	University of Rochester Medical Center, USA
Lianfen Qian	Florida Atlantic University, USA
Qi-Man Shao	Hong Kong University of Science and Technology,
	Hong Kong, China
Zhonggen Su	Zhejiang University, China
Suojin Wang	Texas A&M University, USA
Wensheng Wang	East China Normal University and
	Hangzhou Normal University, China
Zengwu Wang	Academy of Mathematics and Systems Science, China
Baoxue Zhang	Northeast Normal University, China
Li-xin Zhang	Zhejiang University, China
Shurong Zheng	Northeast Normal University, China
Yimin Xiao	Michigan State University, USA
Jia-An Yan	Academy of Mathematics and Systems Science, China
Hailiang Yang	The University of Hong Kong, Hong Kong, China
Jian Yang	Zhejiang University, China
Zhenhai Yang	Beijing University of Technology, China
Hao Yu	University of Western Ontario, Canada
Jun Yu	Swedish University of Agricultural Sciences, Sweden
Jun Zhu	Zhejiang University, China

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