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# Valuation of guaranteed unitized participating life insurance under GEV distribution

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The price of option is affected by high volatilities of asset returns. Normal distribution and geometric Brownian motion cannot characterize leptokurtosis and heavy tails of asset returns, which leads to a biased option pricing. Due to guaranteed unitized participating life insurance contracts typically contain various types of implied options, the contract premium will be significantly biased by distribution assumptions. Considering the economic crisis which may change the distribution, this paper extends valuation method of guaranteed unitized participating life insurance under the generalized extreme value (GEV) distribution. Based on the assumption that the returns follow the GEV distribution, we establish a multi-factor fair valuation pricing model of guaranteed unitized participating life insurance contract. It can explicitly capture the negative skewness and the excess kurtosis of asset returns. We study effects of different factors on embedded option values and calculate different annual premiums. The Least-Squares Monte Carlo simulation method is used to simulate the pricing model. Finally, we compare the parameter sensitivity under the GEV and Normal asset returns.

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## 1. INTRODUCTION

Today participating life insurance products are quite popular in insurance market. The main reason is that they provide the opportunity to link the capital invested into the policy to the performance of a portfolio of equities. In terms of market size, participating life insurance products are considered as the most important modern life insurance products in major insurance and finance markets around the world, such as Britain, America and China, etc. Participating life insurance products are investment plans with associated life insurance benefits, a specified benchmark return, a guarantee of an annual minimum rate of return and a specified rule of the distribution of annual excess investment return above the guaranteed return. These policies are characterized by the fact that the policyholders share the insurer's profits. Wilkie [30] initiated the use of modern option pricing theory to study the embedded options in bonuses on participating life-insurance policies. Grosen and Jørgensen [20] and Ballotta et al. [8] provided a sound research on different contractual features of participating policies. Bacinello [2] considered a minimum guaranteed interest rate to discuss the fair valuation problem of participating policies. Jensen et al. [21] priced surrender options embedded in participating policies with the analysis of guaranteed interest rate and surplus participation. Bacinello [3, 4, 5, 6] concentrated on the analysis of the surrender option in life insurance contracts under mortality risk with single and periodic premiums. Consiglio and De Giovanni [10] evaluated the surrender option within an incomplete markets framework. Gerber [19] calculated the value of equity-linked death benefits in various participating life insurance products. Bacinello [7] supposed the pricing of participating life insurance policies with surrender options using a recursive binomial tree approach.

Due to the fact that participating life insurance contracts typically contain various types of implicit options, option pricing theory is widely employed for pricing insurance contracts. Much attention has been concerned to the valuation of participating life insurance under the Black-Scholes framework, including Tiong [29], Lee [22], Lin et al.[23], Gatzert and Holzmüller [18], Fan [16]. Furthermore, crises, e.g. the 2008 Financial Crisis, have changed the view that extreme events in markets have negligible probabilities. Over the last decade, the situation of life insurance industry has deteriorated due to changes in economic and regulatory environment. It has been a growing shift from modeling "normal" assets to "sharp-heavy" assets that characterize models for extreme events. The Black-Scholes model cannot explicitly explain the negative skewness and the excess kurtosis of returns. Typically, when the left skewness of asset prices increase, the model overprices out-of-the-money call options and underprices in-the-money call options. Melick and Thomas [27] mentioned that it was more natural to begin with an assumption about the future distribution of the underlying asset, rather than the particular stochastic process by which it evolved, and to use option prices to

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directly recover the parameters of that distribution. Therefore, Extreme value theory is a kind of robust framework to analyze the tail behavior of distributions during the financial crisis. Extreme value theory has been applied extensively in various fields especially in the insurance industry, in case of insufficient solvency problem for insurance companies.

The systematic study of extreme value theory for risk management and financial models were done by Embrechts et al. [13, 14, 15] and Mc Neil [26]. Aparicio and Hodges [1] and Corrado [11] studied the option pricing based on generalized beta functions of the second kind and generalized Lambda distribution respectively. Rocco [28] summarized some new developments of extreme value theory for finance. Yang et al. [31] investigated the applications in modeling multivariate long-tailed data by a generalized beta copula. Markose and Alentorn [25] argued that there was difference in pricing options used Generalized Extreme Value (GEV) distribution and Normal distribution for asset returns. The results presented show that the option price was highly sensitive to the changes in the tail shape, which was distinct to its sensitivity to the variance of the return distribution. Cui and Ma [12] studied the pricing synthetic CDO with MGB2 distribution. Ma et al. [24] evaluated the default risk of bond portfolios with extreme value theory.

Existing literatures rarely considered the pricing for participating life insurance products in the case of GEV asset returns. With fluctuations in the financial markets, does the price of participating life insurance products change sharply under the changing financial market? This will be a problem worth studying. Therefore, the framework of [25] is used in our article; we introduce Generalized Extreme Value (GEV) distribution to fit the distribution of asset returns in order to highlight the characteristic features of heavy tail and the skew in asset returns. Then we make a pricing rubric for participating life insurance products embedded the surrender option and analyze the sensitivity of parameters. Finally, we study the different pricing results of participating life insurance products between GEV and Normal asset returns.

The remainder of this paper is structured as follows: In Section 2, we briefly introduce the extreme value theory and Generalized Extreme Value (GEV) distribution. Our valuation framework about four contracts with different embedded options and the annual premium of every contract is explicitly written out in Section 3. We present some numerical results for different values of the involved parameters under GEV distribution for asset returns, including a sensitivity analysis in Section 4. As a comparison, in Section 4 we also discuss the parameters sensitivity under the assumption of Normal distribution for asset returns. Then, a comparison is made about the difference between the two pricing results under two different distributions. Finally, our study has been concluded in Section 5.

## 2. EXTREME VALUE THEORY AND GEV DISTRIBUTION

The role of the generalized extreme value (GEV) distribution in the extreme value theory is analogous to that of the normal distribution in the central limit theory for sums of random variables. Assuming that the underlying random variables $\{X_i, 1 \leq i \leq n\}$ are iid with a finite variance from a distribution $F$ and writing $S_n = \sum_{i=1}^{n} X_i$ for the sum of the first $n$ random variables. The classical central limit theorem (CLT) states that appropriately normalized sums $(S_n - a_n)/b_n$ converges for $n \to \infty$ to a standard normal distribution.

The extreme value problem can be considered as an analogue to the central limit problem. The only possible nondegenerate limiting distributions for normalized block maxima are in the GEV family.

The extreme value distribution arises from the research about the limit theorem of extreme values or maxima in sample data written by [17]. The standardized GEV distribution includes a location parameter $\mu$, a scale parameter $\sigma$ and a tail shape parameter $\xi$. The cumulative distribution functions (c.d.f) would be given by:

$$(1)\quad \begin{aligned} &F(x;\xi,\mu,\sigma) = \exp\left\{-\left[1+\xi\left(\tfrac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}, \\ &1+\xi\tfrac{(x-\mu)}{\sigma} > 0, \sigma > 0, \mu, \xi \in \Re \end{aligned}$$

The subset of the GEV family with $\xi = 0$ is interpreted as the limit of Eq. (1) as $\xi \to \infty$, leading to the Gumbel family with distribution function:

$$(2)\quad F(x;\mu,\sigma) = \exp\left\{-\exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]\right\}, \qquad x \in \Re$$

The probability density functions (p.d.f) can be obtained by

$$(3)\quad \begin{aligned} &f(x;\xi,\mu,\sigma) \\ &= \tfrac{1}{\sigma}\left[1+\xi\left(\tfrac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1}\exp\left\{-\left[1+\xi\left(\tfrac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, \xi \neq 0 \end{aligned}$$

and

$$(4)\quad \begin{aligned} &f(x;\mu,\sigma) \\ &= \tfrac{1}{\sigma}\exp\left[-\left(\tfrac{x-\mu}{\sigma}\right)\right]\exp\left\{-\exp\left[-\left(\tfrac{x-\mu}{\sigma}\right)\right]\right\}, \xi = 0 \end{aligned}$$

The numerical size relation between $\xi$ and 0 determines the type of GEV distributions. The GEV distribution belongs to the Gumbel class when the tail shape parameter $\xi = 0$, which includes the normal, exponential, gamma and lognormal distributions. All moments of the Gumbel class distributions are 0 or finite. When $\xi > 0$, the GEV distribution changes to Fréchet class, which includes some typical heavy tailed distributions such as the Pareto, Student-t and Cauchy distributions. The GEV distribution associated with $\xi < 0$ is called Weibull class, which includes some distributions such as uniform and beta distributions.

## 3. VALUATION MODEL UNDER GEV ASSET RETURNS

### 3.1 Assets value and GEV asset returns model

This article builds a model framework based on the paper of [3, 4]. The aim of the present article is to study the pricing for guaranteed unitized participating life insurance issued at time 0 and maturing $T$ years later. As it is well known, if the insured dies within the term of the contract, it can obtain a specified amount of money (set to $C_1$) and annual dividends according to the policy agreement. Under the contract, the insurer is obliged to pay a terminal dividend if the insured survives until the maturity date $T$. The insurance company would set up a specified fund account for the insured and the initial balance of this account is the annual premium $P$ paid by the insured[1]. These account funds will be increased under a minimum guaranteed yield rate $i$. The annual dividends of the insured are related to the investment performance of the insurance company in each year and the dividends will be accumulated into the insured fund account. The terminal dividend of the insured is related to the overall surplus of the insurance company at the expiration of the contract.

Because the annual dividend and terminal dividend are related to the changes of annual payments, we need to investigate the changes of the assets value. The insured have their own capital accounts for their insurance contracts. The accounts will receive the annual premium $P$ at each policy time and form a new initial value of portfolio combined with the existed portfolio. Assuming that investment strategy does not change due to the inflow of funds and the annual return on assets $g_t$ follows the same distribution. Let $A_t$ denote the value of assets at time $t$ before the inflow of annual premium. $g_t$ represents the rate of return on the reference portfolio in year $t$ of contract. According to the paper of [3], $A_t$ can be expressed as

$$A_t = (A_{t-1} + P) \cdot (1 + g_t) \tag{5}$$

Additionally, the expectation of At is given by

$$E\left(A_t\right) = \left[E\left(A_{t-1}\right) + P\right] \cdot \left[1 + \int g_t f\left(g_t\right) dg_t\right] \tag{6}$$

where, the probability density function (p.d.f) of $g_t$ is denoted as $f(g_t)$. When $P$ is known, the Eq. (6) can be solved iteratively. Then, we can get the expected assets value $E(A_t)$ at maturity.

We denote by ${}_tV$ the insurance policy reserve at time $t$ $(t = 1, \ldots, T-1)$. Assuming the added value of insurance policy reserve ${}_tV$ at the end of the policy year is determined by a certain percentage $z_t$ of the reserve at the beginning of the policy year. Then,

$$_{t+1}V = {}_tV(1 + z_t) = {}_tV + {}_tV z_t \tag{7}$$

$$\Delta({}_{t+1}V) \triangleq {}_{t+1}V - {}_tV = {}_tV z_t \tag{8}$$

Insurance policy reserves will be invested in the financial market, taken as a special portfolio of assets. First introduce the following notation: Annual benefits will be allocated to the insured at a certain percentage which is denoted as $\eta$. Then insurance policy reserves will be increased $\eta g_t$. According to the provisions of the participating life insurance, fund growth rate cannot be less than the minimum guaranteed yield rate $i$. We can obtain some relationships written as follows:

$$(1 + z_t) = \max(1 + i, 1 + \eta g_t) \qquad t = 1, 2, ..., T \tag{9}$$

In fact, the return on assets is assumed to follow the Gumbel class distributions which is a kind of Generalized Extreme Value (GEV) distributions. $E(1 + z_t)$ has an analytical solution that is critical for computing $\pi(C_t)$, the discounted present value of the annual payment. When the return rate $g_t$ follows the GEV distribution of Gumbel family, $E(1 + z_t)$ can be calculate as follows:

$$E\left(1 + z_t\right) = E\left[\max\left(1 + i, 1 + \eta g_t\right)\right], t = 1, 2, ..., T \tag{10}$$

$$\max\left(1 + i, 1 + \eta g_t\right) = \begin{cases} 1 + \eta g_t, & g_t \geq i/\eta \\ 1 + i \quad, & g_t < i/\eta \end{cases} \tag{11}$$

$E(1 + z_t)$ can be written as:

(12)

$$\begin{aligned} E\left(1 + z_t\right) &= \int_{i/\eta}^{\infty} (1 + \eta g_t) f(g_t) dg_t + \int_{-\infty}^{i/\eta} (1 + i) f(g_t) dg_t \\ &= \int_{i/\eta}^{\infty} (1 + \eta g_t) f(g_t) dg_t + (1 + i)\left(1 - \int_{i/\eta}^{\infty} f(g_t) dg_t\right) \\ &= \int_{i/\eta}^{\infty} (\eta g_t - i) f(g_t) dg_t + (1 + i) \end{aligned}$$

Let the p.d.f of $g_t$ with parameters $(\mu, \sigma)$ into the equation above, then we derive the analytical equation of $E(1 + z_t)$:

(13)

$$\begin{aligned} &E\left(1 + z_t\right) \\ &= \left[\eta\sigma \cdot \gamma(0, e^{-h}) + (\eta\mu - i) + (\eta\sigma h - \eta\mu + i)\exp(-e^{-h})\right] + (1 + i) \end{aligned}$$

where, $h = \frac{1}{\sigma}\left(\frac{i}{\eta} - \mu\right)$, $\gamma(0, e^{-h}) = \int_0^{e^{-h}} t^{-1} e^{-t} dt$.

As a comparison, when the return rate gt follows $N(\mu, \sigma)$, $E(1 + z_t)$ exists the analytic solution as follows:

(14)

$$\begin{aligned} &E\left(1 + z_t\right) \\ &\quad = \frac{1}{\sqrt{2\pi}}\left[\eta\sigma\varphi_1 + (\eta\mu - i)\varphi_2\right] + (1 + i) \end{aligned}$$

[1]To simplify the problem statement, assuming that premiums paid annually, while payment term is same to the maturity date.

$$= \frac{1}{\sqrt{2\pi}}\left[\eta\sigma\exp(-\frac{h^2}{2})+(\eta\mu-i)[1-\sqrt{2\pi}N(h)]\right]+(1+i)$$

where $h=\frac{1}{\sigma}\left(\frac{i}{\eta}-\mu\right)$ and $N(\cdot)$ denotes the standard Gaussian distribution.

## 3.2 The structure of the contract

For pricing the guaranteed unitized participating life insurance, we start with a basic endowment insurance contract $B_1$ including an initial guaranteed death benefit. According to the features of different options, three options are included into the basic contract. On the basis of the basic endowment insurance contract, we define a model for a contract $B_2$ added an annual dividend option to $B_1$; a contract $B_3$ added a terminal dividend option to $B_2$; a contract $B_4$ added a surrender option to $B_3$. Finally, the fair valuation of guaranteed unitized participating life insurance contract could be obtained.

### 3.2.1 Basic contract

Firstly, this paper considers a basic endowment insurance contract $B_1$ with periodic premium payments. For the contract $B_1$, if it dies within the term of the contract, the insured will receive the death benefits from the insurer. Meanwhile, the insured will obtain the expected premium payments in the case of survival until maturity. It is assumed that the death benefits are equal to the survival payments. According to the actuarial mathematics theory for an endowment policy, if the age of the insured is $x$, annual premium $P_1$ can be calculated as follows:

$$(15)\qquad \begin{aligned} P_1 &= C_1 P_{x:\,\overline{T}|\mathrm{r}} = C_1 \frac{A_{x:\,\overline{T}|\mathrm{r}}}{\ddot{a}_{x:\,\overline{T}|\mathrm{r}}} \\ &= C_1 \frac{\sum_{t=1}^{T-1}(1+r)^{-t}\,{}_{t-1|}q_x + (1+r)^{-T}\,{}_{T-1}p_x}{\sum_{t=0}^{T-1}(1+r)^{-t}\,{}_t p_x} \end{aligned}$$

where, $C_1$ denotes death or survival payments, $r$ is used as the annual interest rate for discounting future benefits and premiums. ${}_{t-1|}q_x$ represents the probability that the insured dies between $t-1$ and $t$ year of contract, and ${}_t p_x$ is the probability for an $x$ year old policyholder surviving for the next $t$ years.

Corresponding to the annual premium $P_1$, the single premium $U^{P_1}$ can be written as

(16)
$$\begin{aligned} U^{P_1} &= C_1 A_{x:\,\overline{T}|\mathrm{r}} \\ &= C_1\left[\sum_{t=1}^{T-1}(1+r)^{-t}\,{}_{t-1|}q_x + (1+r)^{-T}\,{}_{T-1}p_x\right] \end{aligned}$$

The single premium can be used to calculate the value of annual dividend option.

### 3.2.2 Contract with annual dividend option

Next, on the basic features of the basic contract $B_1$, we include an annual dividend option and call this contract $B_2$. The annual premium is set as $P_2$.

The expected value of the benefits to the insured will be increased by annual dividend. Set annul benefits for the insured as $C_t$, then $C_t = C_1 + \textit{annual dividends}$. By $C_t$ is the benefit payable at time $t$ ($t = 2,\dots,T$), we need consider the insurance policy reserve of this contract at the end of each policy year $t$ in order to calculate $C_t$. The insurance policy reserve ${}_tV$ can be calculated according to the actuarial equivalence principle for the endowment policy at time $t$ ($t = 1,\dots,T-1$):

(17)
$$\begin{aligned} {}_tV &= C_t A_{x+t:\,\overline{T-t}|\mathrm{r}} - P_2 \ddot{a}_{x+t:\,\overline{T-t}|\mathrm{r}} \\ &= C_t\left[\sum_{h=1}^{T-t}(1+r)^{-h}\,{}_{h-1|}q_{x+t} + (1+r)^{-T}\,{}_{T-t}p_{x+t}\right] \\ &-P_2\sum_{h=0}^{T-t-1}(1+r)^{-h}\,{}_h p_{x+t} \\ &t = 1,2,\dots,T-1 \end{aligned}$$

Eq. (17) is just a theoretical formula because $C_t$ and $P_2$ are both unknown among the equation. Hence, ${}_tV$ cannot be calculated directly and we need to use Eq. (8) to price the contract with an annual dividend option.

Combining and solving Eq. (17) and Eq. (8), we can deduce the expression of annual benefits $C_t$:

(18)
$$C_t = C_1\left\{\prod_{k=1}^{t-1}(1+z_k) - \sum_{k=1}^{t-1}\left[z_k\left(1-\frac{k}{T}\right)\prod_{h=k+1}^{t-1}(1+z_h)\right]\right\},\quad t = 2,3,\dots,T$$

The detailed proof of Eq. (17) can be seen at the article written by [3]. In order to calculate the annual equilibrium premium of contract $B_2$, supposing financial market risks and mortality risks are independent, we assume that the risks of insurance companies are neutral. On the basis of these two assumptions, the specific calculation is implemented in two steps. Firstly, the expected value of the benefit $C_t$ is discounted to the beginning of the insurance period, and denote $\pi(C_t)$ as discounted present value. Considering the expected value of the death risk, by $\pi(C_t)$ we can calculate the single premium $U^{P_2}$. Secondly, $P_2$ can be calculated according to the actuarial principle between $U^{P_2}$ and $P_2$.

At time $t = 1$, one gets

$$(19)\qquad \pi(C_1) = C_1(1+r)^{-1}$$

At time $t > 1$,

$$(20)\qquad \begin{aligned} \pi(C_1) &= E\left[C_1(1+r)^{-t}\right] \\ &= C_1(1+r)^{-t}\left\{\prod_{k=1}^{t-1}E(1+z_k)\right. \\ &\left.-\sum_{k=1}^{t-1}\left[\left(1-\frac{k}{T}\right)E(z_k)\prod_{h=k+1}^{t-1}E(1+z_h)\right]\right\} \end{aligned}$$

Under the framework of GEV asset returns model, if the return rate $g_t$ follows the GEV distribution of Gumbel family, taking Eq. (13) into Eq. (20), $\pi(C_t)$ can be calculated

*Table 1. The balance sheet of participating life insurance*

| Point of time | Asset | Liability and equity |
|---|---|---|
| $T$ | $A_T$ | ${}_TV$ |

as:

$$\pi(C_t) = C_1(1+r)^{-t}\left\{\prod_{k=1}^{t-1}\{[\eta\sigma\cdot\gamma(0,e^{-h})+\eta\mu-i) + (\eta\sigma h-\eta\mu+i)\exp(-e^{-h})]+I\} - \sum_{k=1}^{t-1}\left[(1-\frac{k}{T})E(z_k)\prod_{h=k+1}^{t-1}\{[\eta\sigma\cdot\gamma(0,e^{-h}) + u)+(\eta\sigma h-u)\exp(-e^{-h})]+I\}\right]\right\} \tag{21}$$

where, $h=\frac{1}{\sigma}\left(\frac{i}{\eta}-\mu\right)$, $\gamma(0,e^{-h})=\int_0^{e^{-h}} t^{-1}e^{-t}dt$, $u=\eta\mu-i$, $I=1+i$.

Then, the expression of $U^{P_2}$ under GEV asset returns is given by:

$$U^{P_2}=\sum_{t=1}^{T-1}\pi(C_t)_{\,t-1|}q_x+\pi(C_T)_{T-1}p_x \tag{22}$$

Then, the annual premium P2 under GEV asset returns is given by:

$$P_2=\frac{U^{P_2}}{\ddot{a}_{x:\,\overline{T}|r}}=\frac{\sum_{t=1}^{T-1}\pi(C_t)_{\,t-1|}q_x+\pi(C_T)_{T-1}p_x}{\ddot{a}_{x:\,\overline{T}|r}} \tag{23}$$

$U^{P_2}$ minus $U^{P_1}$ is equal to the value of annual dividend option. Moreover, the average annual value of annual dividend option can be expressed as the difference between $P_2$ and $P_1$.

### 3.2.3 Contract with terminal dividend option

In addition to the features of the contract $B_2$ with annual dividend option, we include a terminal dividend option called contract $B_3$. Let $P_3$ and $U^{P_3}$ denote the annual premium and the single premium. The terminal dividends are related to the surplus of the insurance company at maturity $T$. Then, the balance sheet of the guaranteed unitized participating life insurance at time $T$ is shown in Table 1.

At the expired date of contract, the value of $A_T$ and ${}_TV$ will determine whether the policyholder can get the terminal dividend. If $A_T$ less than ${}_TV$, it means that the value of the investment assets cannot cover the balance of the insured's capital account (${}_TV$ is equal to $C_T$, actually). The insured can only obtain $A_T$. When $A_T$ is greater than ${}_TV$, there are surpluses in the capital account. Therefore, the insured will receive the final dividend, which is a certain proportion $\beta$ of surpluses. The terminal dividend of the insured can be expressed as $\beta(A_T-C_T)^+$.

Considering the annual dividend, the actual benefit payable $C'_T$ of contract $B_3$ at time $T$ is given by

$$C'_T=\begin{cases} A_T \quad , & A_T\le C_T \\ C_T+\beta(A_T-C_T)^+ \quad , & A_T>C_T \end{cases} \tag{24}$$

Assume that the assets obey the GEV distribution. Then, combined with the formulation of single premium about $B_2$, the single premium $U^{P_3}$ of $B_3$ can be written as

$$\begin{aligned} U^{P_3} &= \sum_{t=1}^{T-1}\pi(C_t)_{\,t-1|}q_x+\pi(C'_T)_{T-1}p_x \\ &= U^{P_2}+[\pi(C'_T)-\pi(C_T)]_{T-1}\,p_x \end{aligned} \tag{25}$$

where, $\pi(C'_T)=E\left[C'\cdot(1+r)^{-t}\right]$.

The annual premium $P_3$ can be written as

$$P_3=\frac{\sum_{t=1}^{T-1}\pi(C_t)_{\,t-1|}q_x+\pi(C'_T)_Tp_x}{\ddot{a}_{x:\,\overline{T}|r}} \tag{26}$$

The difference $U^{P_3}-U^{P_2}=[\pi(C'_T)-\pi(C_t)]_{\,T}p_x$ in Eq. (25) is the value of terminal dividend option. Whereas the difference between $P_3$ and $P_2$ is the average annual value of terminal dividend option under the GEV asset returns model.

### 3.2.4 Contract with surrender option

Finally, we consider the framework on the basis of contract $B_3$ and include a surrender option named contract $B_4$, which the policyholder can exercise annually at time $t=1,2,\ldots,T$ until maturity. When exercising the surrender option at time $\tau$, the policyholder can obtain the insurance policy reserve ${}_\tau V$ at time of termination of the contract $B_4$. The contract payoff at time of surrender $t=\tau$ can be written out as follows: The annual premium $P_3$ can be written as

$$ {}_\tau V=C_\tau A_{x+\tau:\,\overline{T-\tau}|r}-P_3\ddot{a}_{x+\tau:\,\overline{T-\tau}|r}\ ,\quad t=1,2,...,T-1 \tag{27}$$

The insured will select an optimal admissible exercise strategy according to some information at time $\tau$, which includes the assets value, reserve, risk-free interest rate, etc. For the insured, the optimal surrender exercise time is the moment that the insured can obtain the maximum expected value of cash flow. The expected total discounted payoff of cash flow acquired by the policyholder after exercising the surrender option is given by

$$ {}_\tau U^{P_4}=\sum_{t=1}^{\tau}\pi(C_t)_{\,t-1|}q_x+\pi({}_\tau V)_\tau p_x \tag{28}$$

The single premium $U^{P_4}$ of contract $B_4$ with surrender option is denoted by

$$U^{P_4} = E\left[\sup_{\tau\in\Gamma[0,T-1]}\left({}_\tau U^{P_4}\right)\right] \tag{29}$$

We denote by $\Gamma[0,T-1]$ all the values at stopping time from 0 to $T-1$. The annual premium $P_4$ of contract $B_4$ can be written as

$$P_4 = \frac{E\left[\sup_{\tau\in\Gamma[0,T-1]}\left({}_\tau U^{P_4}\right)\right]}{\ddot{a}_{x:\,\overline{T}|r}} \tag{30}$$

$U^{P_4}$ minus $U^{P_3}$ is equal to the value of surrender option, while the difference between $P_4$ and $P_3$ is the average annual value of surrender option.

# 4. SIMULATION UNDER GEV DISTRIBUTION

## 4.1 Assets value simulation and model solution

In order to price the guaranteed unitized participating life insurance, we need to get the expected assets value $E(A_t)$ at the end of each policy year. According to the Eq. (6), the calculation of $E(A_t)$ relates to the distribution of the return on assets and annual premium $P$. We suppose that the return on assets follow the GEV distribution and the expectation of annual return rate can be simulated after having estimated the corresponding parameters. Actually, $E(A_t)$ is used to calculate the terminal dividend, which existed at contract $B_3$ and $B_4$, whereas other contracts can be calculated only by the annual return rate of the assets value. Hence, when pricing contract $B_4$, we use the annual premium $P_3$ of contract $B_3$ to simulate the expected value of assets in each policy year. Similarly, pricing contract $B_3$, we use the annual premium $P_2$ of contract $B_2$. The contract $B_2$ uses the annual premium $P_1$ of contract $B_1$ to simulate the expected value of assets.

In order to price the contract $B_4$ with three options mentioned above, we should simulate from $B_1$ to $B_4$. Firstly, according to the actuarial principle, we calculate the single premium and annual premium of contract $B_1$. Secondly, we assume return on assets following the GEV distribution, and then calculate the single premium and annual premium of contract $B_2$. Thirdly, based on the average annual premium of contract $B_2$, we calculate the single premium and annual premium of contract $B_3$. Finally, we can calculate the single premium and annual premium of contract $B_4$ by the Least-Squares Monte Carlo simulation (LSM) method. The difference of single premium between contracts is the value of embedded option. Specifically, the value of annual dividend is equal to $U^{P_2}$ - $U^{P_1}$; the value of terminal dividend is equal to $U^{P_3}$ - $U^{P_2}$; the value of surrender dividend is equal to $U^{P_4}$ - $U^{P_3}$.

## 4.2 Simulation and numerical results

For numerical computation, we need to set the parameters involved in the previous sections. The mortality is used in this study was derived from the experience life table of China Life Insurance (2000–2003). The experience life table includes four groups of mortality for Chinese people. Because the impact of pricing on the mortality of different populations is not the focus of this article, we use the non-pension male mortality as the initial parameter. Then the initial age of the insured is set as 40 years old. The basic payment of guaranteed unitized participating life insurance $C_1$ is set as 10000 yuan and the insurance period is 10 years. The scheduled interest rate of China Life Insurance Policy fell to 2.5% in 1999 and has maintained this level so far. Therefore, the minimum guaranteed yield rate $i$ is set at 2.5%. The risk-free interest rate $r$ is set at the current one-year fixed deposit rate 1.5%. The ratio of annual dividend and final dividend, according to the provisions of the CIRC (China Insurance Regulatory Commission), are set at 75% and 50% respectively. Finally, there are three parameters $\xi, \mu, \sigma$ belonging to the GEV distribution. In this study, we use the CSI 300 stock index data from April 2005 to December 2015 to estimate these three parameters and the results are $\xi = -0.2744, \mu = -0.0187, \sigma = 0.0976$.

Using the parameters we set above, the average annual premium and single premium of contract from $B_1$ to $B_4$ can be calculated respectively as follows:

$U^{P_1} = 8625.97, U^{P_2} = 10764.24, U^{P_3} = 11188.46, U^{P_4} = 11319.74$

$P_1 = 927.76, P_2 = 1157.74, P_3 = 1203.37, P_4 = 1217.49$

Then, according to the single premium of contract from $B_1$ to $B_4$, the value of embedded option can be calculated. The value of annual dividend is equal to $U^{P_2} - U^{P_1} = 2138.27$; the value of terminal dividend is equal to $U^{P_3} - U^{P_2} = 424.22$; the value of surrender dividend is equal to $U^{P_4} - U^{P_3} = 131.28$.

From the above pricing results about three kinds of embedded options, the value of the annual dividend is the highest and its value accounted for 24.79% of single premium of guaranteed unitized participating life insurance. However, the value of surrender option is the lowest and its value accounted for only 1.17% of single premium of contract $B_3$.

In order to compare the difference of pricing results between the GEV distribution and Normal distribution, the basic parameters are set as same as the parameters followed the GEV distribution except the parameters of the distribution, which are $x = 40, T = 10, C_1 = 10000, i = 2.5\%, r = 1.5\%, \eta = 75\%, \beta = 50\%$. For the parameters of Normal distribution are estimated by the data of the CSI 300 stock index data from April 2005 to December 2015. The results are $\mu = 0.0155, \sigma = 0.0961$.

As comparison, using the parameters we set above, when the return rate $g_t$ follows the Normal distribution, the single premium of contract from $B_1$ to $B_4$ can be calculated respectively as follows:

*Table 2. The premiums with respect to the various parameters*

| | $x$ | $i$ | $r$ | $\eta$ | $\beta$ | $\xi$ | $\sigma$ | $\mu$ |
|---|---|---|---|---|---|---|---|---|
| $P_1$ | $\nearrow$ | $=$ | $\searrow$ | $=$ | $=$ | $=$ | $=$ | $=$ |
| $A$ | $\searrow$ | $\nearrow$ | $\searrow$ | $\nearrow$ | $=$ | $\nearrow$ | $\nearrow$ | $\nearrow$ |
| $P_2$ | $\nearrow$ | $\nearrow$ | $\searrow$ | $\nearrow$ | $=$ | $\nearrow$ | $\nearrow$ | $\nearrow$ |
| $T$ | $\nearrow$ | $\nearrow$ | $\searrow$ | $=$ | $\nearrow$ | $\nearrow$ | $\nearrow$ | $\nearrow$ |
| $P_3$ | $\nearrow$ | $\nearrow$ | $\searrow$ | $\nearrow$ | $\nearrow$ | $\nearrow$ | $\nearrow$ | $\nearrow$ |
| $S$ | $\searrow$ | $\searrow$ | $\nearrow$ | $\nearrow$ | $\nearrow$ | $\searrow\nearrow$ | $\nearrow$ | $\nearrow\searrow$ |
| $P_4$ | $\nearrow$ | $\nearrow$ | $\searrow$ | $\nearrow$ | $\nearrow$ | $\nearrow$ | $\nearrow$ | $\nearrow$ |

Note: $P_1$ represents the price of basic contract $B_1$, annual dividend option $A$, $P_2 = P_1 + A$ represents the price of contract $B_2$, terminal dividend option $T$, $P_3 = P_2 + T$ represents the price of contract $B_3$, surrender option $S$, $P_4 = P_3 + S$ represents the price of contract $B_4$.

$U^{P_1} = 8625.97, U^{P_2} = 10705.30, U^{P_3} = 11098.63, U^{P_4} = 11229.29$

The corresponding average annual premium would be given by:

$P_1 = 927.76, P_2 = 1151.41, P_3 = 1193.71, P_4 = 1207.76$

Then, according to the single premium of contract from $B_1$ to $B_4$, the value of embedded option can be calculated. The value of annual dividend is equal to $U^{P_2} - U^{P_1} = 2079.33$; the value of terminal dividend is equal to $U^{P_3} - U^{P_2} = 393.33$; the value of surrender dividend is equal to $U^{P_4} - U^{P_3} = 130.66$.

Compared with the pricing results, the return rate $g_t$ follows the GEV distribution, it can be seen that when the return rate $g_t$ follows the Normal distribution the values of these three embedded options are lower than the three values with the rate return followed the GEV distribution. Taking into account the distribution of return rate with the characteristics of "asymmetric" and "heavy tail" in the capital market, pricing the contracts by using the return rate following the Normal distribution with the characteristics of "normal" and "light tail" will underestimate the value of embedded options.

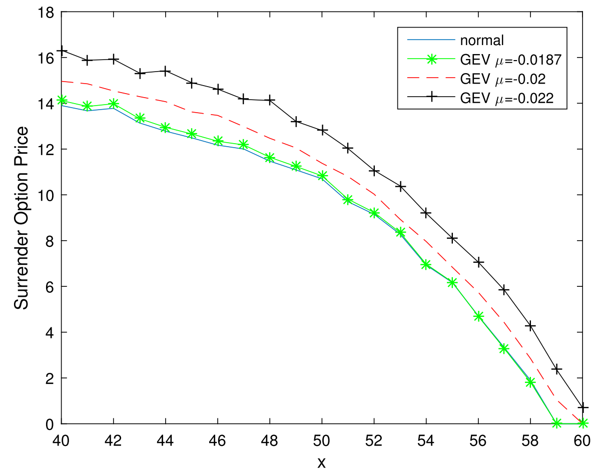


*Figure 1. The average annual premium of surrender option versus the age of the insured $x$ under two different distributions.*

## 4.3 Results comparisons between GEV and normal asset returns

In order to investigate the influence of the parameters on pricing results, the method of parameter sensitivity analysis was carried out on the basis of the initial parameters under both GEV and Normal asset returns. The results in Table 2 illustrate the expected behavior of the premiums with respect to the various parameters.

Then, we compare the values of embedded surrender options under two different assumptions of the distribution of the return rate.

The results in Table 3 and Figure 1 illustrate the average annual premium of surrender option and its proportion of annual premium with age increased per 2 years from 40 to 60 years old under two different distribution of the return rate. The value of surrender option decreases with growing

*Table 3. The average annual premium of surrender option versus the age of the insured $x$ under two different distributions (in $RMB$ terms)*

| $x$ | GEV distribution ($\mu = -0.0187$) | | Normal distribution | |
|---|---|---|---|---|
| | Average annual premium | Proportion | Average annual premium | Proportion |
| 40 | 14.12 | 1.1734% | 13.90 | 1.1641% |
| 42 | 13.98 | 1.1615% | 13.78 | 1.1533% |
| 44 | 12.95 | 1.0733% | 12.78 | 1.0670% |
| 46 | 12.34 | 1.0199% | 12.17 | 1.0138% |
| 48 | 11.64 | 0.9609% | 11.47 | 0.9537% |
| 50 | 10.83 | 0.8918% | 10.70 | 0.8873% |
| 52 | 9.23 | 0.7569% | 9.15 | 0.7559% |
| 54 | 6.97 | 0.5686% | 6.93 | 0.5696% |
| 56 | 4.69 | 0.3805% | 4.71 | 0.3849% |
| 58 | 1.82 | 0.1466% | 1.90 | 0.1539% |
| 60 | 0.00 | 0.0000% | 0.00 | 0.0000% |

Note: Average annual premium $= P_4 - P_3$; Proportion $= (P_4 - P_3)/P_3$.

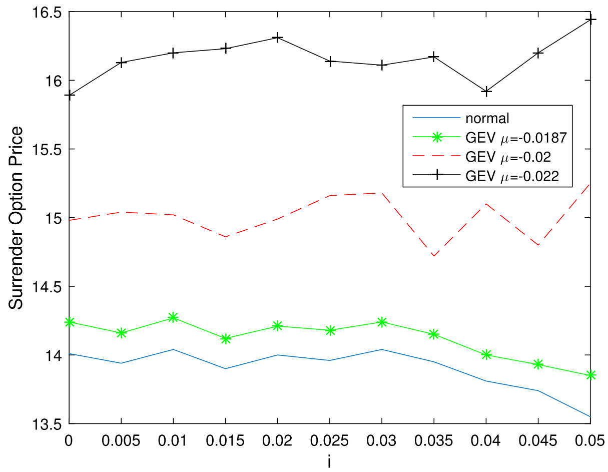


*Figure 2. The average annual premium of surrender option versus the minimum guaranteed yield rate $i$ under two different distributions.*

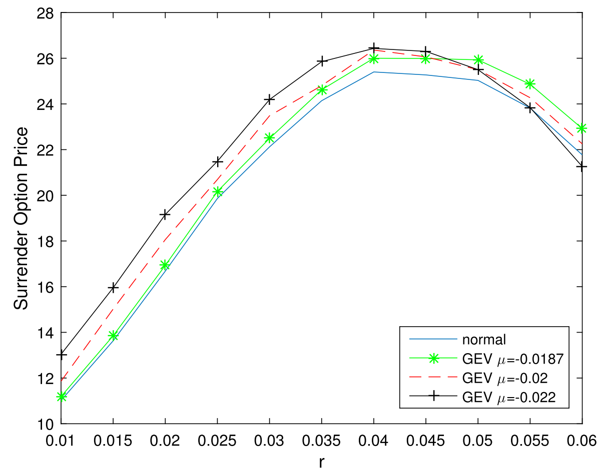


*Figure 3. The average annual premium of surrender option versus the risk-free interest rate $r$ under two different distributions.*

$x$. In most cases, the average annual premium of surrender option and its proportion under the GEV distribution of the return rate is higher than the result under the Normal distribution. But with the increase of age $x$, the difference of average annual premium under GEV distribution and Normal distribution is gradually reducing. When the age of 56 years old is reached, the average annual premium under assumption of Normal distribution is even higher than it is under assumption of GEV distribution. This shows that when the insurance companies price the contract for the low age of the insured, the effect of distribution of rate return on the value of surrender option need to be taken seriously. In addition, with the mean of GEV distribution further deviating from Normal distribution, the price of the embedded options would be changed substantially.

The results in Table 4 and Figure 2 illustrate the average annual premium of surrender option and its proportion of annual premium for different values of minimum guaranteed yield rate $i$ with step 0.005 varying between 0 and 0.05 under two different distributions of the return rate. The average annual premium of surrender option fluctuates with growing $i$, showed a trend of descension in general. The proportions of annual premium under two different distributions also show a downtrend. Concerned about the differences of average annual premium of surrender option under these two hypotheses of distributions, we can see that the gap of the value is stable. When other parameters fixed, only changing the minimum guaranteed yield rate cannot reduce the value of embedded surrender option under two distributions of the return rate. In addition, with the mean of GEV distribution further deviating from Normal distribution, the price of the embedded options would be changed significantly.

The results in Table 5 and Figure 3 illustrate the average annual premium of surrender option and its proportion of

*Table 4. The average annual premium of surrender option versus the minimum guaranteed yield rate $i$ under two different distributions (in $RMB$ terms)*

| $i$ | GEV distribution ($\mu = -0.0187$) | | Normal distribution | |
|---|---|---|---|---|
| | Average annual premium | Proportion | Average annual premium | Proportion |
| 0 | 14.24 | 1.2550% | 14.01 | 1.2437% |
| 0.005 | 14.16 | 1.2327% | 13.94 | 1.2231% |
| 0.01 | 14.27 | 1.2313% | 14.04 | 1.2205% |
| 0.015 | 14.12 | 1.2028% | 13.90 | 1.1935% |
| 0.02 | 14.21 | 1.1967% | 14.00 | 1.1880% |
| 0.025 | 14.18 | 1.1783% | 13.96 | 1.1696% |
| 0.03 | 14.24 | 1.1688% | 14.04 | 1.1606% |
| 0.035 | 14.15 | 1.1436% | 13.95 | 1.1361% |
| 0.04 | 14.00 | 1.1156% | 13.81 | 1.1093% |
| 0.045 | 13.93 | 1.0924% | 13.74 | 1.0852% |
| 0.05 | 13.85 | 1.0688% | 13.55 | 1.0529% |

Note: Average annual premium $= P_4 - P_3$; Proportion $= (P_4 - P_3)/P_3$.

*Table 5. The average annual premium of surrender option versus the risk-free interest rate $r$ under two different distributions (in $RMB$ terms)*

| $r$ | GEV distribution ($\mu = -0.0187$) | | Normal distribution | |
|---|---|---|---|---|
| | Average annual premium | Proportion | Average annual premium | Proportion |
| 0.01 | 11.19 | 0.8892% | 11.02 | 0.8821% |
| 0.015 | 13.85 | 1.1508% | 13.63 | 1.1419% |
| 0.02 | 16.93 | 1.4726% | 16.68 | 1.4621% |
| 0.025 | 20.17 | 1.8399% | 19.86 | 1.8224% |
| 0.03 | 22.49 | 2.1110% | 22.12 | 2.0869% |
| 0.035 | 24.62 | 2.3749% | 24.14 | 2.3415% |
| 0.04 | 26.00 | 2.5784% | 25.40 | 2.5327% |
| 0.045 | 25.99 | 2.6497% | 25.27 | 2.5902% |
| 0.05 | 25.93 | 2.7180% | 25.03 | 2.6385% |
| 0.055 | 24.86 | 2.6802% | 23.82 | 2.5819% |
| 0.06 | 22.92 | 2.5405% | 21.77 | 2.4265% |

Note: Average annual premium = $P_4 - P_3$; Proportion = $(P_4 - P_3)/P_3$.

*Table 6. The average annual premium of surrender option versus the annual dividend ratio $\eta$ under two different distributions (in $RMB$ terms)*

| $\eta$ | GEV distribution ($\mu = -0.0187$) | | Normal distribution | |
|---|---|---|---|---|
| | Average annual premium | Proportion | Average annual premium | Proportion |
| 0.20 | 11.77 | 1.0825% | 11.72 | 1.0804% |
| 0.25 | 12.07 | 1.1036% | 12.00 | 1.1005% |
| 0.30 | 11.91 | 1.0796% | 11.83 | 1.0764% |
| 0.35 | 12.15 | 1.0927% | 12.07 | 1.0900% |
| 0.40 | 12.08 | 1.0753% | 11.99 | 1.0719% |
| 0.45 | 12.39 | 1.0939% | 12.29 | 1.0908% |
| 0.50 | 12.75 | 1.1147% | 12.62 | 1.1099% |
| 0.55 | 13.03 | 1.1283% | 12.90 | 1.1237% |
| 0.60 | 12.84 | 1.0993% | 12.70 | 1.0942% |
| 0.65 | 13.55 | 1.1502% | 13.37 | 1.1429% |
| 0.70 | 13.81 | 1.1594% | 13.62 | 1.1518% |
| 0.75 | 13.81 | 1.1473% | 13.60 | 1.1393% |
| 0.80 | 14.63 | 1.2043% | 14.40 | 1.1947% |
| 0.85 | 14.93 | 1.2166% | 14.68 | 1.2064% |
| 0.90 | 14.81 | 1.1929% | 14.52 | 1.1797% |
| 0.95 | 15.64 | 1.2473% | 15.33 | 1.2342% |
| 1.00 | 16.18 | 1.2763% | 15.83 | 1.2613% |

Note: Average annual premium = $P_4 - P_3$; Proportion = $(P_4 - P_3)/P_3$.

annual premium for different values of the risk-free interest rate $r$ with step 0.005 varying between 0.01 and 0.06 under two different distributions of the return rate. With the increase of the risk-free interest rate $r$, the annual premium and its proportion shows a trend of first increasing and then decreasing under the two assumptions of the distributions. The differences of average annual premium of surrender option under these two hypotheses of distributions increase gradually with the increase of the risk-free interest rate. It shows that under the condition of high risk-free interest rate the value of surrender option cannot be ignored anymore. Meanwhile, it is quite important to select a proper distribution of the return rate to price the surrender option.

The results in Table 6 and Figure 4 illustrate the average annual premium of surrender option and its proportion of annual premium for different values of the annual dividend ratio $\eta$ with step 0.05 varying between 0.2 and 1.0 under two different distributions of the return rate. The average annual premiums of surrender options and their proportion increase with growing $\eta$ under both two different distributions of the return rate. The average annual premium under GEV distribution is higher than it is under Normal distribution obviously, and the difference increases with the annual dividend ratio $\eta$ by weaker growth. However, with the mean of GEV distribution further deviating from Normal distribution, the price of the embedded options would be changed significantly.

The results in Table 7 and Figure 5 illustrate the average annual premium of surrender option and its proportion of annual premium for different values of the terminal dividend

*Table 7. The average annual premium of surrender option versus the terminal dividend ratio $\beta$ under two different distributions (in $RMB$ terms)*

| $\beta$ | GEV distribution ($\mu = -0.0187$) | | Normal distribution | |
|---|---|---|---|---|
| | Average annual premium | Proportion | Average annual premium | Proportion |
| 0.20 | 13.61 | 1.1758% | 13.40 | 1.1637% |
| 0.30 | 13.52 | 1.1586% | 13.30 | 1.1476% |
| 0.40 | 14.05 | 1.1864% | 13.86 | 1.1787% |
| 0.50 | 14.12 | 1.1727% | 13.91 | 1.1649% |
| 0.60 | 14.04 | 1.1480% | 13.82 | 1.1398% |
| 0.70 | 14.17 | 1.1427% | 13.99 | 1.1373% |
| 0.80 | 14.15 | 1.1235% | 13.94 | 1.1171% |
| 0.90 | 14.28 | 1.1169% | 14.06 | 1.1099% |

Note: Average annual premium = $P_4 - P_3$; Proportion = $(P_4 - P_3)/P_3$.

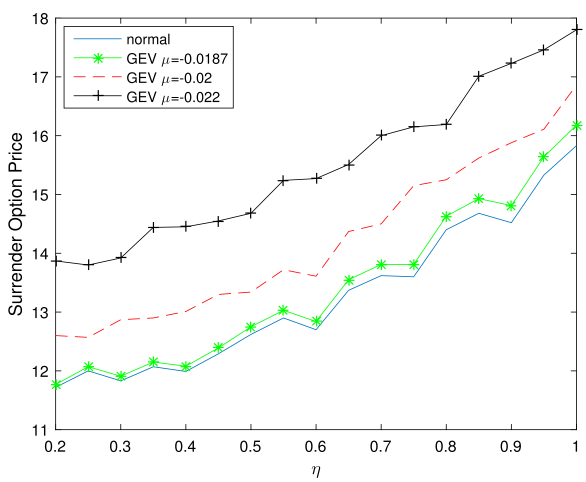


*Figure 4. The average annual premium of surrender option versus the annual dividend ratio $\eta$ under two different distributions.*

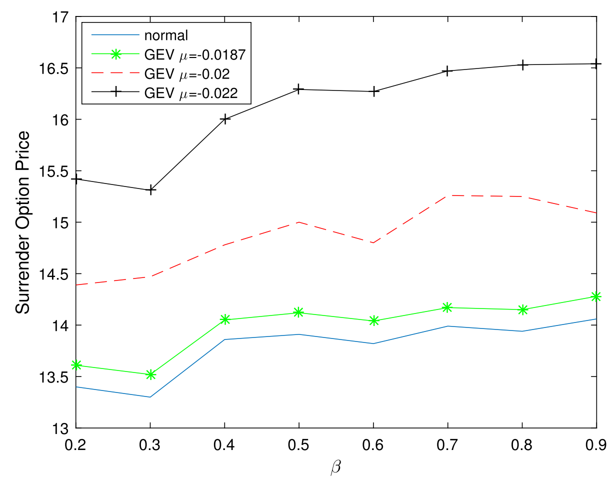


*Figure 5. The average annual premium of surrender option versus the terminal dividend ratio $\beta$ under two different distributions.*

ratio $\beta$ under two different distributions of the return rate. Under the assumption of Normal distribution, the average annual premium of surrender option and its proportion of annual premium are insensitive to $\beta$. The differences of average annual premium of surrender option under these two hypotheses of distributions are relatively stable, but the differences of the proportion are slight decreasing. Similarly, with the mean of GEV distribution further deviating from Normal distribution, the price of the embedded options would be changed greatly.

Overall, by comparing the results of parameter sensitivity analysis under both GEV and Normal distribution of the return rate, the value of surrender option under the hypothesis of GEV distribution is higher than it is under the Normal distribution in most cases. Meanwhile, the changing trends of parameter sensitivity are basically similar under two different distributions. When other parameters are fixed, the gap of the average annual premium of surrender option between GEV distribution and Normal distribution increases gradually with some specific parameters (such as the risk-free interest rate, the annual dividend ratio and the minimum guaranteed yield rate).

It shows that when the parameters are at a high level, it is very critical to select the appropriate distribution to fit the return rate, which can price accurately for the value of surrender option. Especially in the financial crisis and other risk events, GEV distribution can better characterize the volatility of return on assets over expansion-recession cycles, which can improve the precision of embedded option pricing in a participating contract.

## 5. CONCLUDING REMARKS

According to the characteristics of guaranteed unitized participating life insurance, the death rate, surrender and minimum guaranteed yield rate dividend policy are considered in this paper. From the results of study on pricing of guaranteed unitized participating life insurance embedded surrender options, the value of surrender option is sensitive to the pricing parameters including the minimum guaranteed yield rate, the annual dividend ratio and the risk-free interest rate when the return rate follows the GEV distribution assumption. Therefore, when the insurance company is creating insurance plans, they need to reasonably determine the value of these pricing parameters in order to prevent a high number of surrender options resulting in increasing in surrender rates. Through the comparison of the results of parameters sensitivity analysis under GEV distribution and Normal distribution hypothesis, we get the conclusion that the value of surrender option under GEV distribution hypothesis is higher than it is under Normal distribution hypothesis in most cases. Although the changing trends of parameters sensitivity about surrender options under two different distribution hypotheses are basically the same, there is a gap of the value of surrender options between GEV distribution hypothesis and Normal distribution hypothesis under different parameters setting. Taking into account the distribution of return rate with the characteristics of "asymmetric" and "heavy tail" in the capital market, pricing the contracts by using the return rate following the Normal distribution with the characteristics of "normal" and "light tail" will underestimate the value of embedded options. Therefore, it is critical to apply an appropriate distribution of asset return rate for valuing the model precisely.

From analysis above, it is important for stakeholders of the guaranteed unitized participating life insurance to pay attention to these few things: 1) The risk of surrender in crisis time is greater than that in normal time. It protrudes the characteristics of guaranteed participating life insurance, especially when the insured are older age. 2) The values of embedded options and various insurance premiums are very sensitive to the market interest rate and annual dividend ratio. It shows the fact that the value of embedded options needs to be shown separately in the assessment of the fair value of life insurance liabilities. 3) If the annual dividend ratio is too high then the value of surrender option will increase which will cause more surrender events. Therefore, the annual dividend ratio should not be too high. 4) The insurance company can try to improve the terminal dividend ratio as much as possible, on the one hand, it can effectively maintain the average annual premium, and on the other hand, it almost has no effect on the surrender option price. 5) For insurance companies, it requires the insurance company to use their assets in higher risk assets if the balance of guaranteed participating life insurance premium is much higher than the basic participating life insurance premiums. On the opposite, the current dividend of guaranteed participating life insurance premium cannot be higher than the basic participating life insurance too much; otherwise, the price will be excessive. 6) The CIRC must regulate the risk of the asset investment of the insurance company to limit insurance companies' assets use channels and their corresponding proportions. If investment tends to have too much risk, then there is basically no return on the maturity of the policy therefore the corresponding surrender rate will be high, and the corresponding surrender option price will also be high. This is not conducive to protect the interests of the insured.


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